

# Advanced Digital Logic Design – EECS 303

<http://ziyang.eecs.northwestern.edu/eecs303/>

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# Outline

1. Review of state minimization
2. Registers and counters
3. Asynchronous finite state machines

# Minimization of incompletely specified FSMs

CS	NS(0)	NS(1)	OUT
A	A	X	1
B	B	C	1
C	C	A	X
D	A	D	0

## Reason for prime compatibles

Consider the following maximal compatibles

AB

BC

CD

BE  $\rightarrow$  BC

# Minimization stages

- State table
- Implication chart
- Maximal cliques (for larger problems)
  - Largest fully connected subgraphs
- Maximal compatibles
- Prime compatibles
- Binate covering

# Outline

1. Review of state minimization
2. Registers and counters
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# Registers and counters

- Once you understand flip-flops and FSM design, registers and counters are easy
- Shift registers can shift contents left or right
- Registers
  - Commonly a group of D flip-flops written and read simultaneously
- Counters
  - FSMs that have only a clock input
  - Can count up or down in some binary number system
  - Can also cyclicly shift a one through flip-flops (ring counter)

## Multiple-output pseudo-NFAs

- Similar to standard NFAs
- Have multiple accept states
- Simple translation to Moore machines
- Going from DFAs to Mealy machines is more complicated

# Outline

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## Section outline

### 3. Asynchronous finite state machines

Synchronous vs. asynchronous design

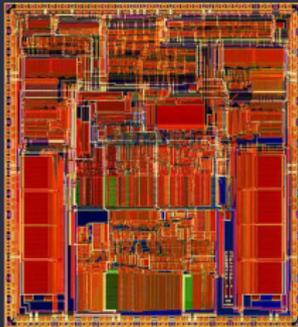
State assignment

State variable synthesis

# Synchronous vs. asynchronous design

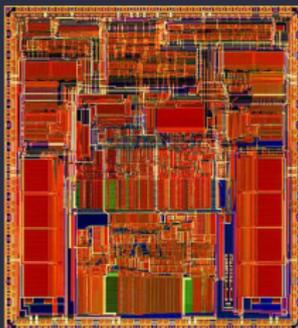
- Synchronous design makes a lot of problems disappear
- Glitches not fatal
- FSM design easier
- However, things are likely to change soon

# Current SOC clocking and communication



**design**

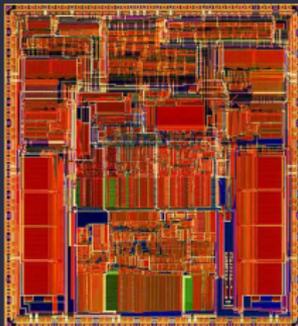
# Future SOC clocking and communication



**design**

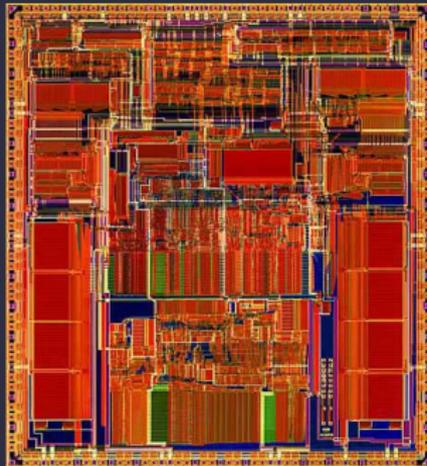
**clock propagation range**

# Future SOC clocking and communication



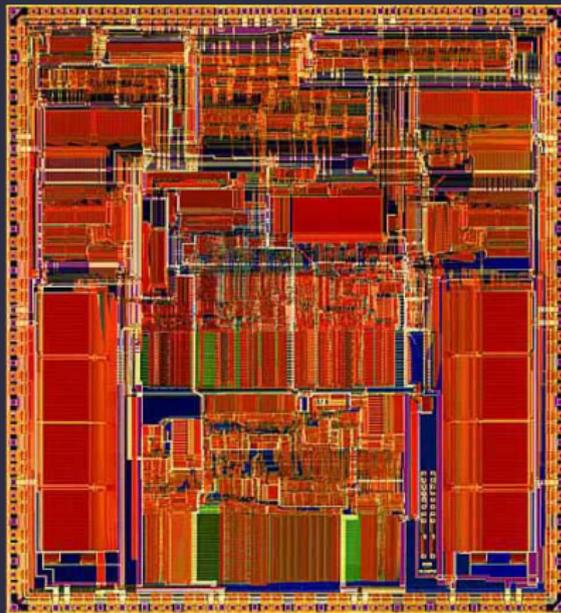
**design**

# Future SOC clocking and communication



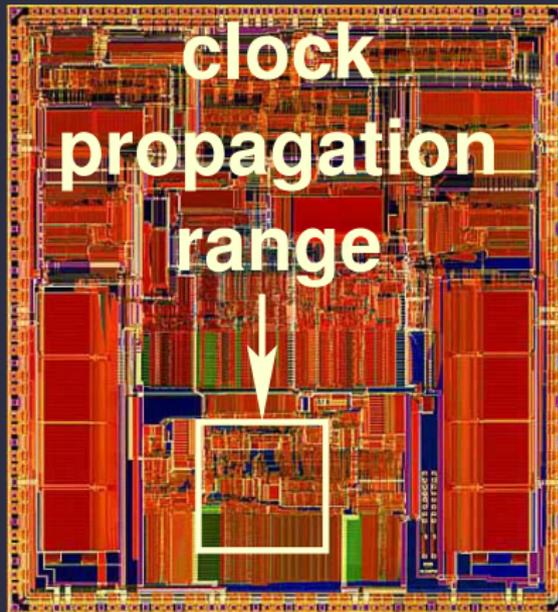
**design**

# Future SOC clocking and communication



# design

# Future SOC clocking and communication



clock  
propagation  
range

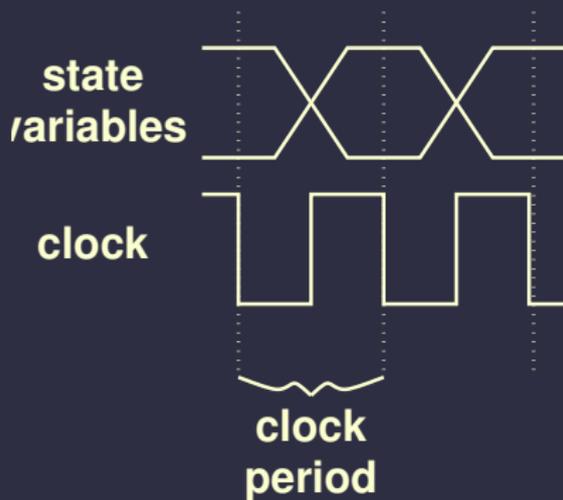
design

# Globally asynchronous, locally synchronous (GALS)

- Complete flexibility in region frequencies
- Reputation for inefficient communication
- However, results always improving
- Asynchronous circuits traditionally skipped
- However, you will encounter them in interface circuits and are likely to encounter them more and more frequently
- Asynchronous design likely to become increasingly important

# Synchronous vs. asynchronous FSMs

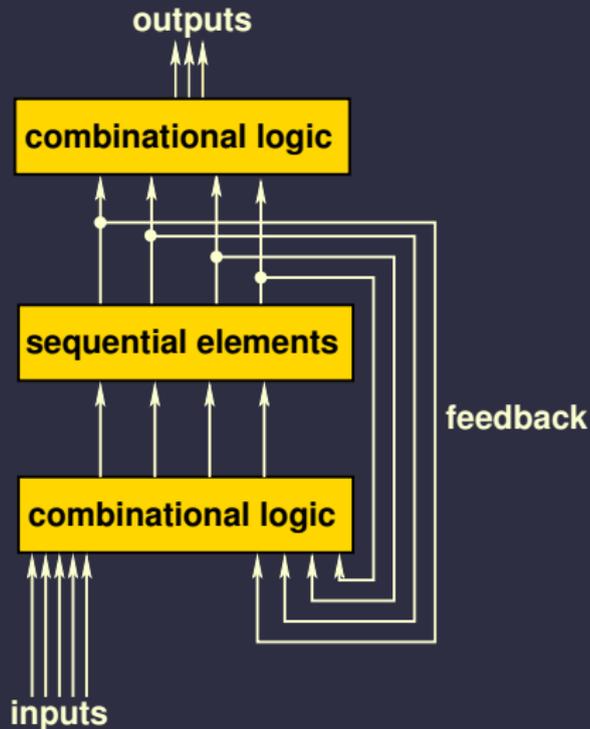
## synchronous FSM



## asynchronous FSM



# Synchronous system



## Differences from synchronous circuits

- Avoid critical races (more later)
- Avoid glitches
- State can be a function of input as well as state variables
- May need to do state splitting

# Asynchronous machine block diagram

**outputs**



**feedback**



**combinational logic**

**inputs**



## Section outline

### 3. Asynchronous finite state machines

Synchronous vs. asynchronous design

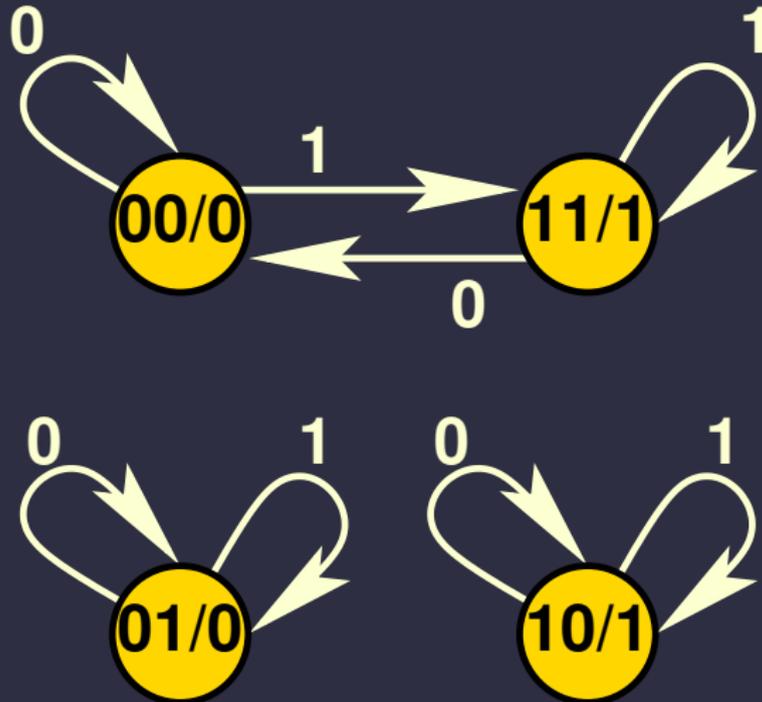
State assignment

State variable synthesis

# Asynchronous FSM state assignment

- For synchronous FSMs, state assignment impacts area and power consumption
- For asynchronous FSMs, incorrect state assignment results in incorrect behavior
- A *race* is a condition in which the behavior of the circuit is decided by the relative switching speeds of two state variables
- An asynchronous FSM with races will not behave predictably
- Avoid *critical races*, races which result in different end states depending on variable change order

# Incorrect asynchronous assignment



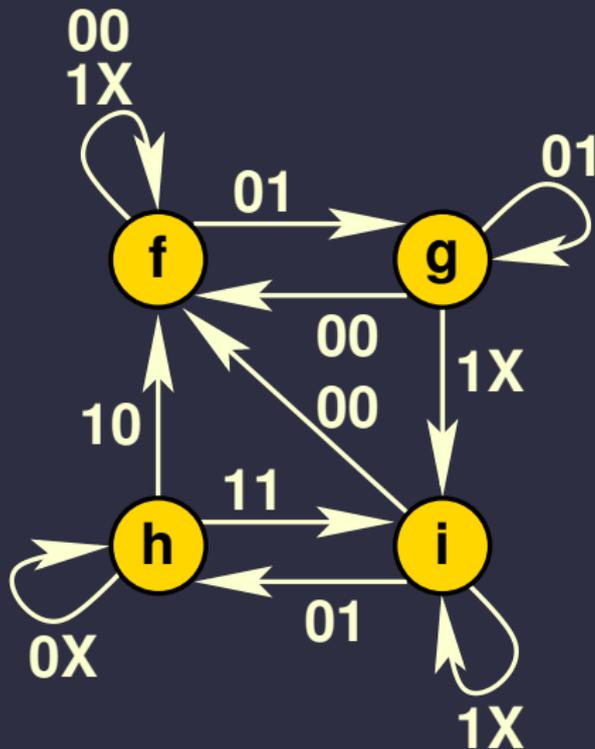
# Asynchronous FSM state assignment

s	s <sup>+</sup>		Q
	0	1	
00	00	11	0
01	01	01	0
10	10	10	1
11	00	11	1

Consider 00 → 11 transition

- Becomes trapped in 01 or 10
- Which one?
  - Random

## Asynchronous FSM adjacency



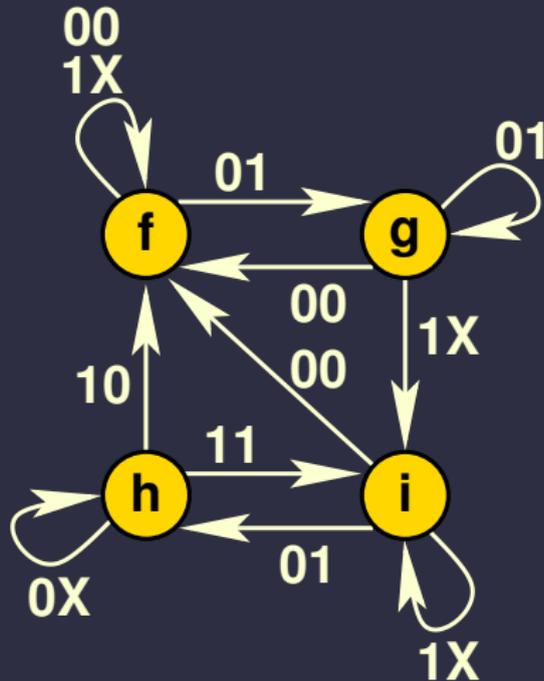
# Asynchronous FSM adjacency

- Two input bits
- When a particular input leads to a state, maintaining that input should generally keep one in the state
  - E.g., 01 for  $g$
- Will show exception later

## Asynchronous FSM adjacency

- $f$  adjacent to  $g$ ,  $h$ , and  $i$
- $g$  adjacent to  $f$  and  $i$
- $h$  adjacent to  $f$  and  $i$
- $i$  adjacent to  $f$ ,  $g$ , and  $h$
- Four states  $\rightarrow \lceil \lg(4) \rceil = 2$  state variables
- However, in 2D space, each point is adjacent to only two others
- Need at least 3D

# Asynchronous FSM adjacency

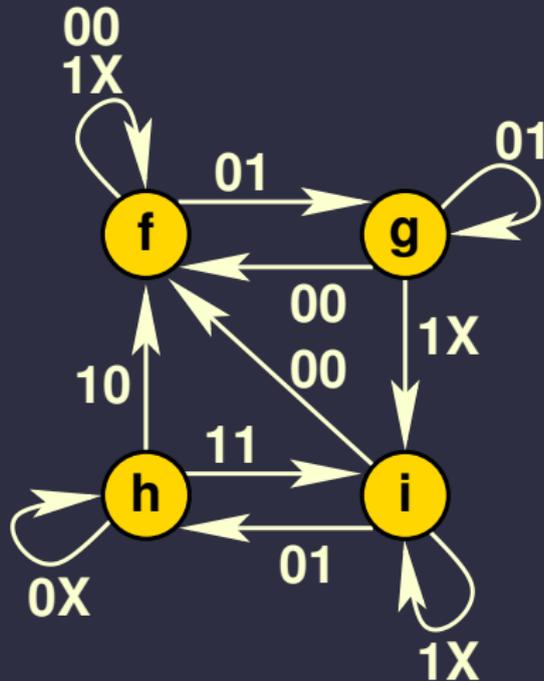


		a	
		0	1
b	0	f	g
	1	h	i

## Asynchronous FSM adjacency

- Need all adjacent states in AFSM to be adjacent
- $i$  to  $f$  transition could be trapped in  $g$ !
- What to do for a graph with too many connections?
- Split states and hop through some states to reach others

# Asynchronous FSM adjacency



	a	
	0	1
b	0	f g
	1	h i

	ab			
	00	01	11	10
c	0	g f	i2 i2	
	1	X h1	h2	X

# Asynchronous FSM adjacency

current state	next state			
	00	01	10	11
f	f	g	f	f
g	f	g	i <sub>2</sub>	i <sub>2</sub>
h <sub>1</sub>	h <sub>1</sub>	h <sub>1</sub>	f	h <sub>2</sub>
h <sub>2</sub>	h <sub>2</sub>	h <sub>2</sub>	h <sub>1</sub>	i <sub>1</sub>
i <sub>1</sub>	f	h <sub>2</sub>	i <sub>1</sub>	i <sub>1</sub>
i <sub>2</sub>	i <sub>1</sub>	i <sub>1</sub>	i <sub>2</sub>	i <sub>2</sub>

# AFSM synthesis redundancy

- Even if AFSM has a fully connected adjacent state assignment there are still additional complications
- State variables must have stable transitions
- E.g., for a SOP implementation, every state pair that is connected in the state transition graph must be covered by at least one cube
- Hazards may cause incorrect operation for AFSMs

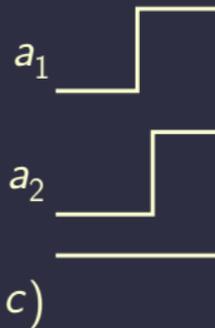
# AFSM transition stability

Given that  $f(a, b, c)$  is a state variable

$f(a, b, c)$		$ab$			
		00	01	11	10
$c$	0	1	1	1	0
	1	0	0	1	0

$$\bar{a} \bar{c} + a b + b \bar{c}$$

$$\bar{a} b \bar{c} \rightarrow a b \bar{c}$$



# AFSM design summary

- AFSMs immediately react to input changes
- No need to worry about clock
- However, design more complicated
- Stability
- Unstable states must have appropriate (no glitches) outputs
- Adjacent states must have adjacent assignments
- Glitches on state variables may be fatal

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# Debouncing

- Recall previous method of debouncing switch
- Consider SPDT (single pole, double throw) switch
- Pull-up/Pull-down resistors?
- Latches?

## Glitches on state variables in AFSMs

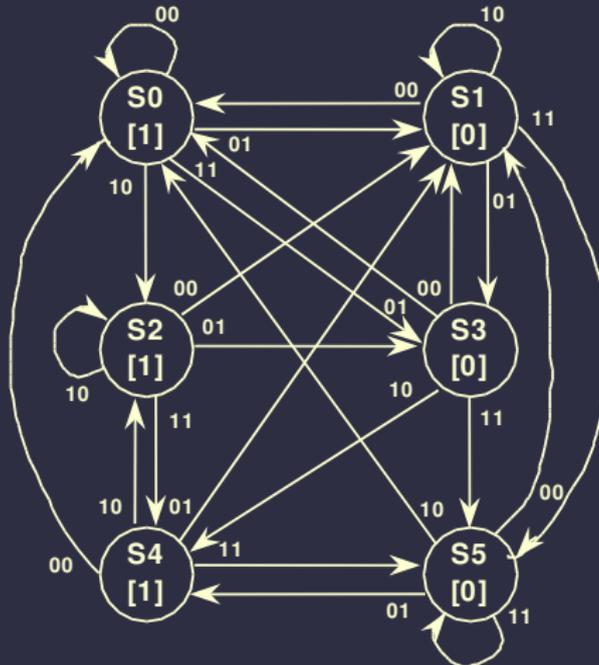
- Can use latches in similar way
- Additional advantage: Separate sequential and combinational logic. feedback requirements reduced
- Disadvantage: May require more logic
- Consider the example

# AFSM design example

- Design a two-input machine (LM)
  - Output 1 iff L is low and M was high at some time during most recent L high period
  - Output 0 otherwise
  - Let's build two AFSMs to solve this problem
  - One will use global feedback
  - One will use RS latches

# Multiple input example

Initial multiple input FSM state diagram

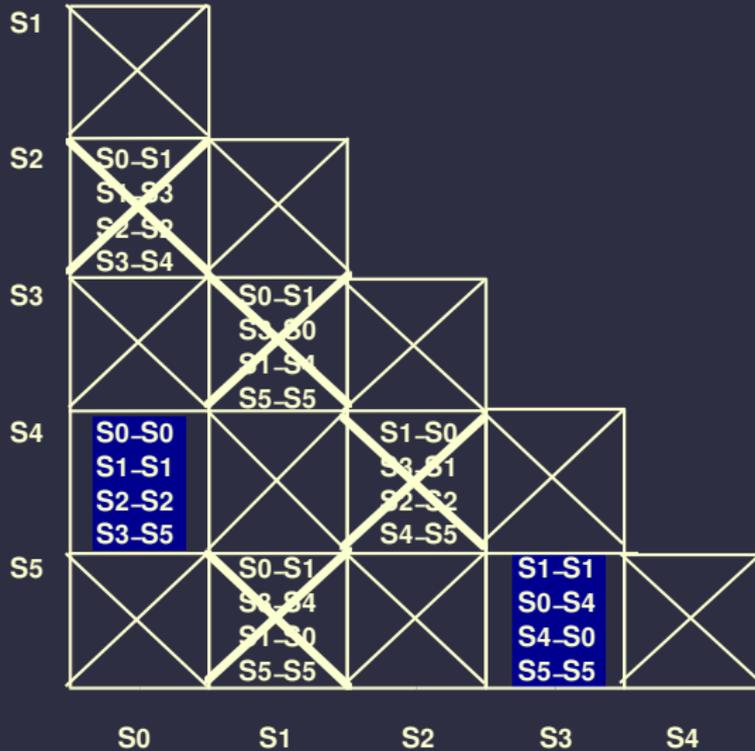


## State Diagram

# State table

Present State	Next State				Output
	00	01	10	11	
S <sub>0</sub>	S <sub>0</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	1
S <sub>1</sub>	S <sub>0</sub>	S <sub>3</sub>	S <sub>1</sub>	S <sub>5</sub>	0
S <sub>2</sub>	S <sub>1</sub>	S <sub>3</sub>	S <sub>2</sub>	S <sub>4</sub>	1
S <sub>3</sub>	S <sub>1</sub>	S <sub>0</sub>	S <sub>4</sub>	S <sub>5</sub>	0
S <sub>4</sub>	S <sub>0</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>5</sub>	1
S <sub>5</sub>	S <sub>1</sub>	S <sub>4</sub>	S <sub>0</sub>	S <sub>5</sub>	0

# Implication chart



## Simplified state table

Present State	Next State				Output
	00	01	10	11	
$S_6$	$S_6$	$S_1$	$S_2$	$S_7$	1
$S_1$	$S_6$	$S_7$	$S_1$	$S_7$	0
$S_2$	$S_1$	$S_7$	$S_2$	$S_6$	1
$S_7$	$S_1$	$S_6$	$S_6$	$S_7$	0