Advanced Digital Logic Design – EECS 303

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Outline

Section outline

1. Transistors in digital systems
   - Switches
   - CMOS
   - AND and OR are harder than NAND and NOR
   - Transmission gates

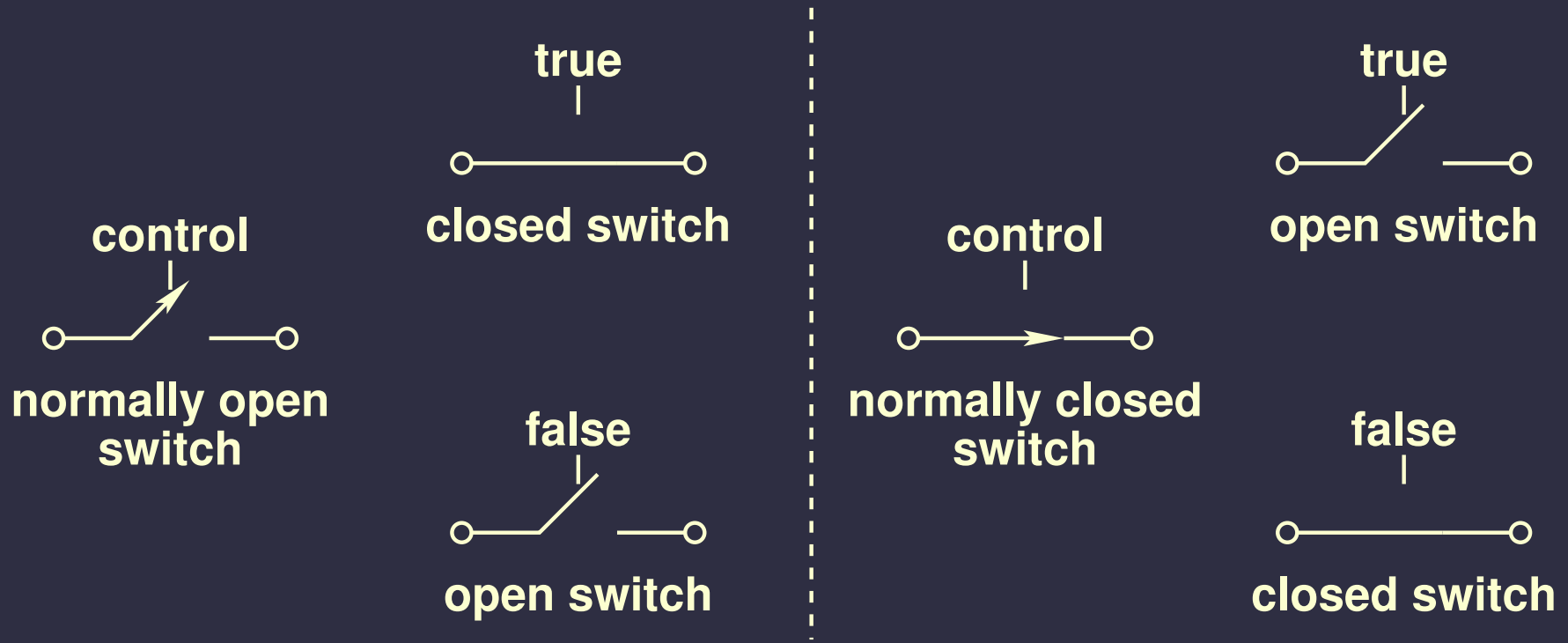
Switch-based design representation

- A switch shorts or opens two points dependant on a control signal
- Used as models for digital transistors
- Why is using normally open and normally closed particularly useful for CMOS?

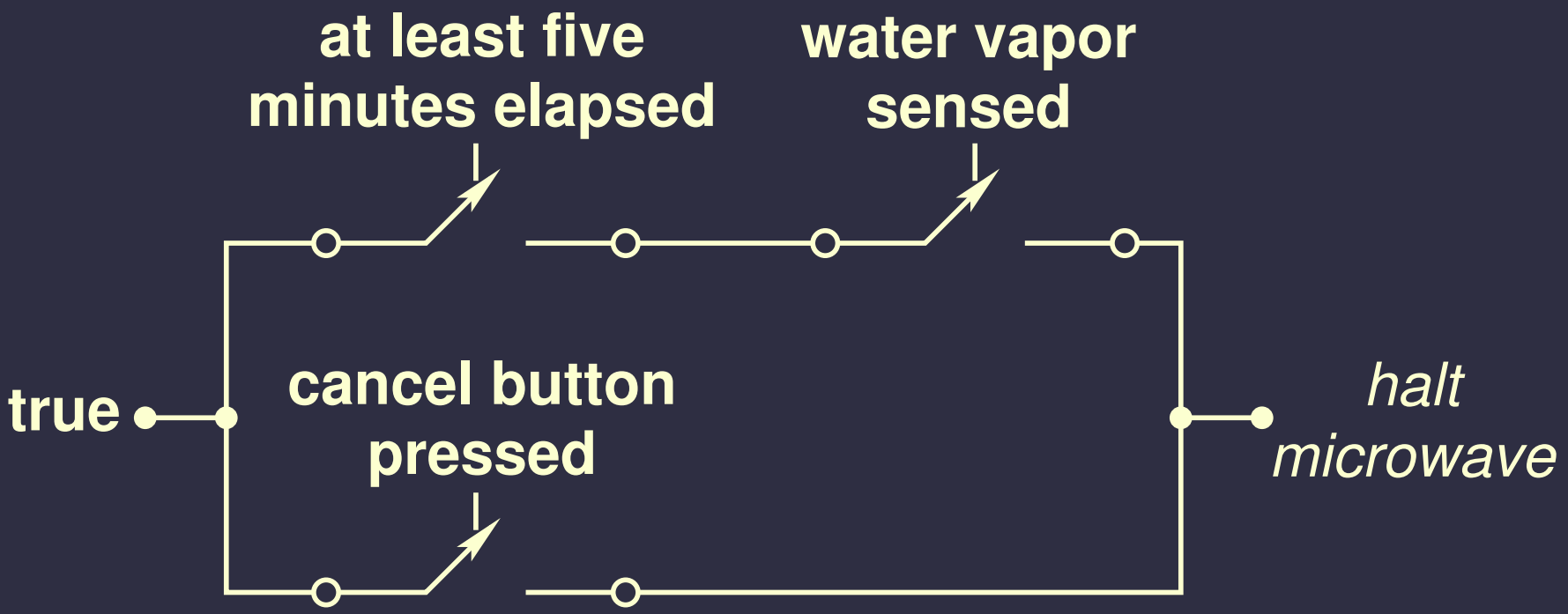
Switch-based design representation

- A switch shorts or opens two points dependant on a control signal
- Used as models for digital transistors
- Why is using normally open and normally closed particularly useful for CMOS?
  - NMOS and PMOS transistors easy to model

Switch-based definitions

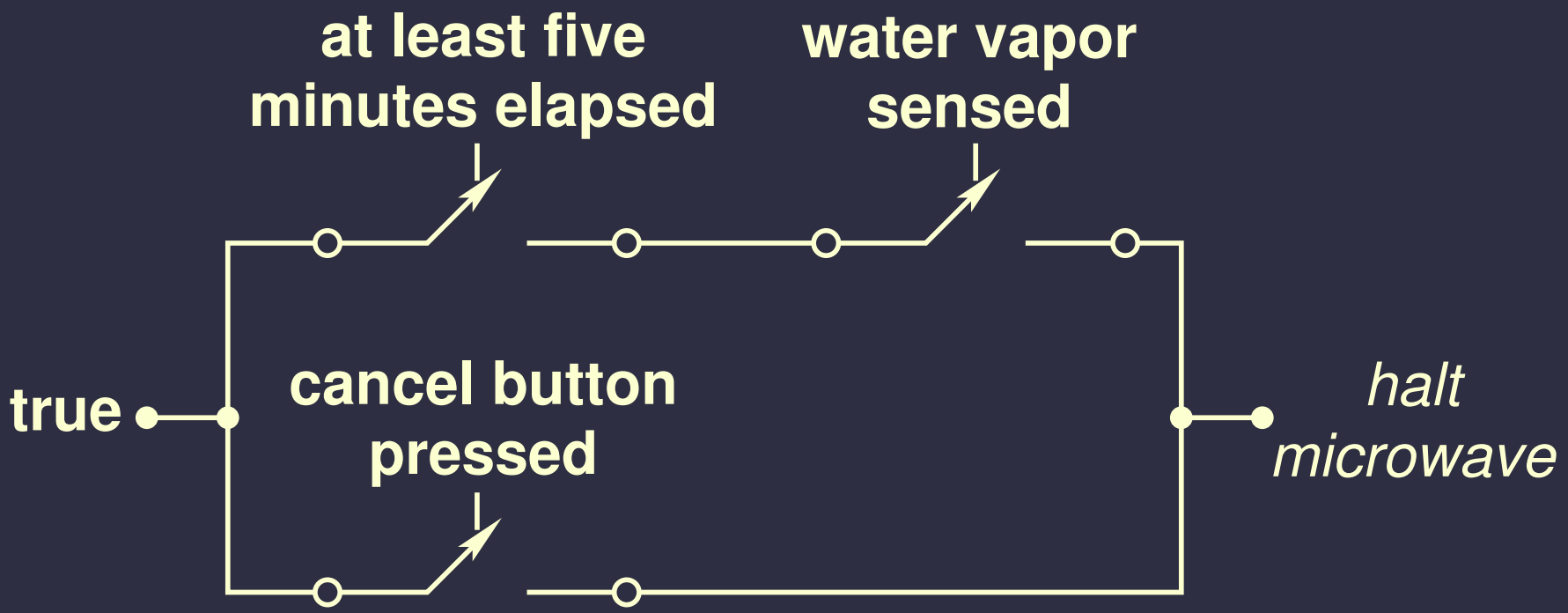


Microwave control example



- What happens if the cancel button is not pressed and five minutes haven't yet passed?

Microwave control example



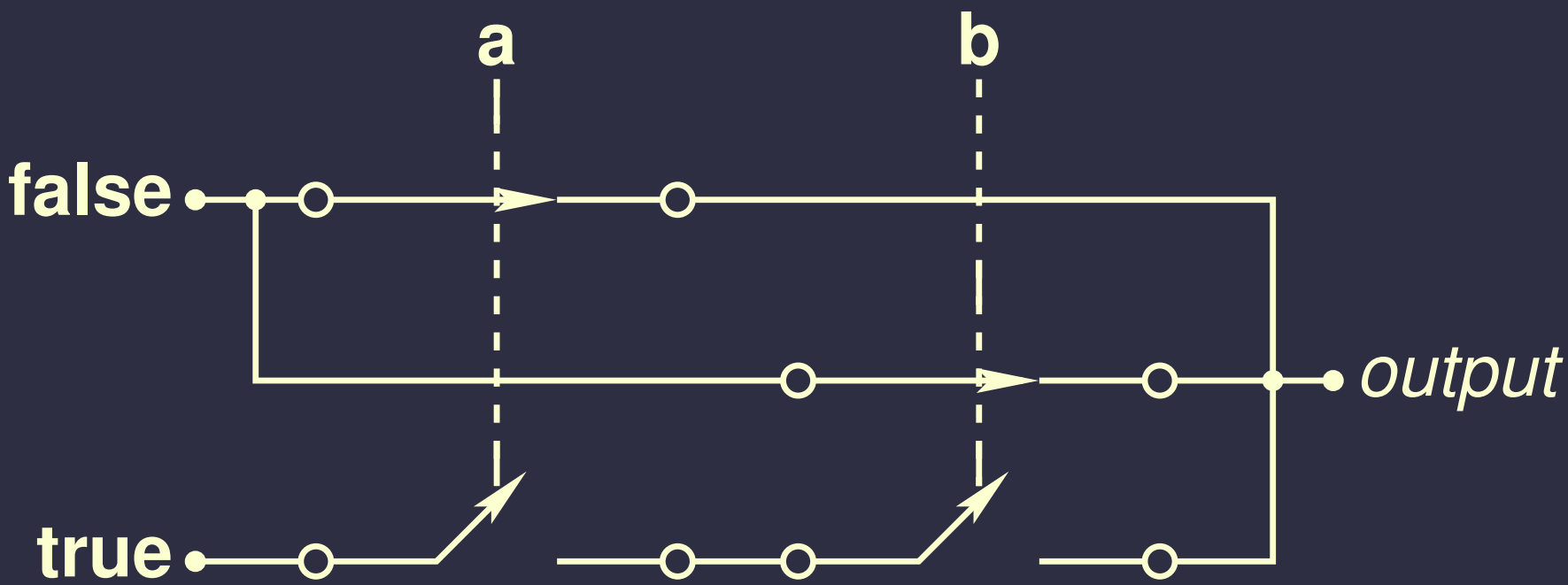
- What happens if the cancel button is not pressed and five minutes haven't yet passed?
  - The output value is undefined.

Constraints on network output

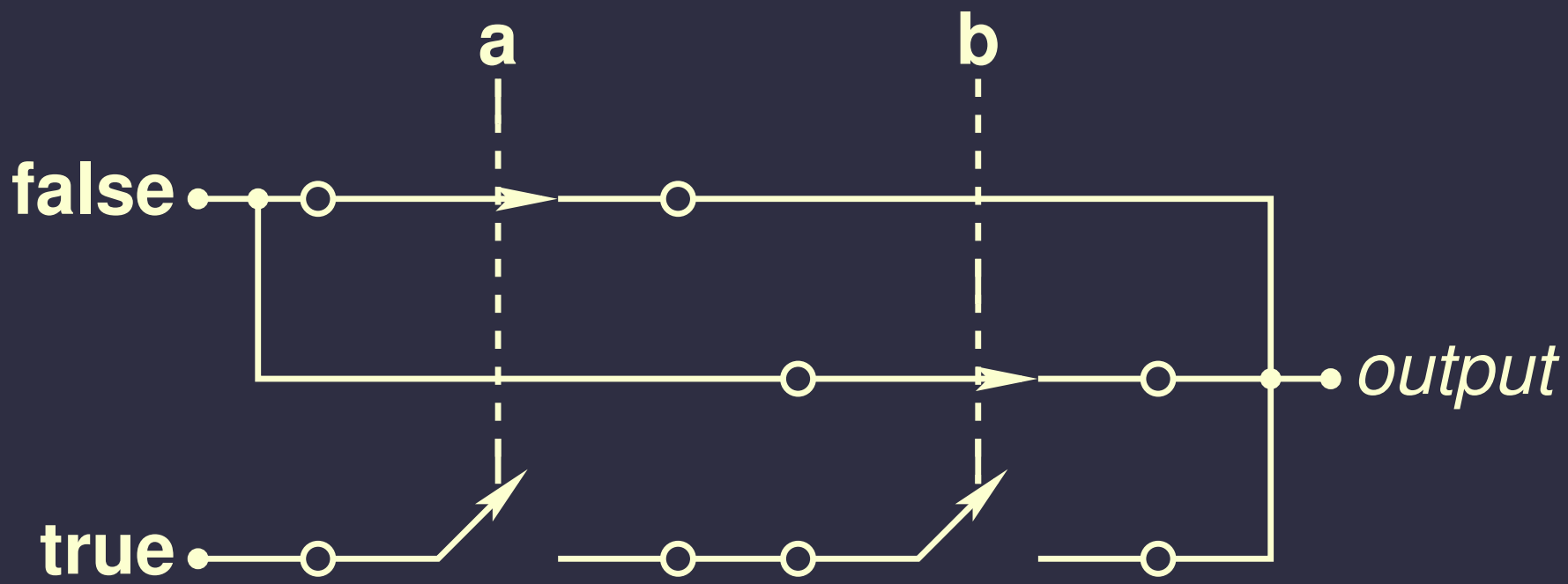
Under all possible combinations of input values

- Each output must be connected to an input value
- No output may be connected to conflicting input values

Switch-based *AND*



Switch-based *AND*



Note that this requires

- Normally closed switches that transmit false signals well
- Normally open switches that transmit true signals well

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   - AND and OR are harder than NAND and NOR
   - Transmission gates

Relationship with CMOS

- Metal Oxide Semiconductor
- Positive and negative carriers
- Complimentary MOS
- PMOS gates are like normally closed switches that are good at transmitting only true (high) signals
- NMOS gates are like normally open switches that are good at transmitting only false (low) signals

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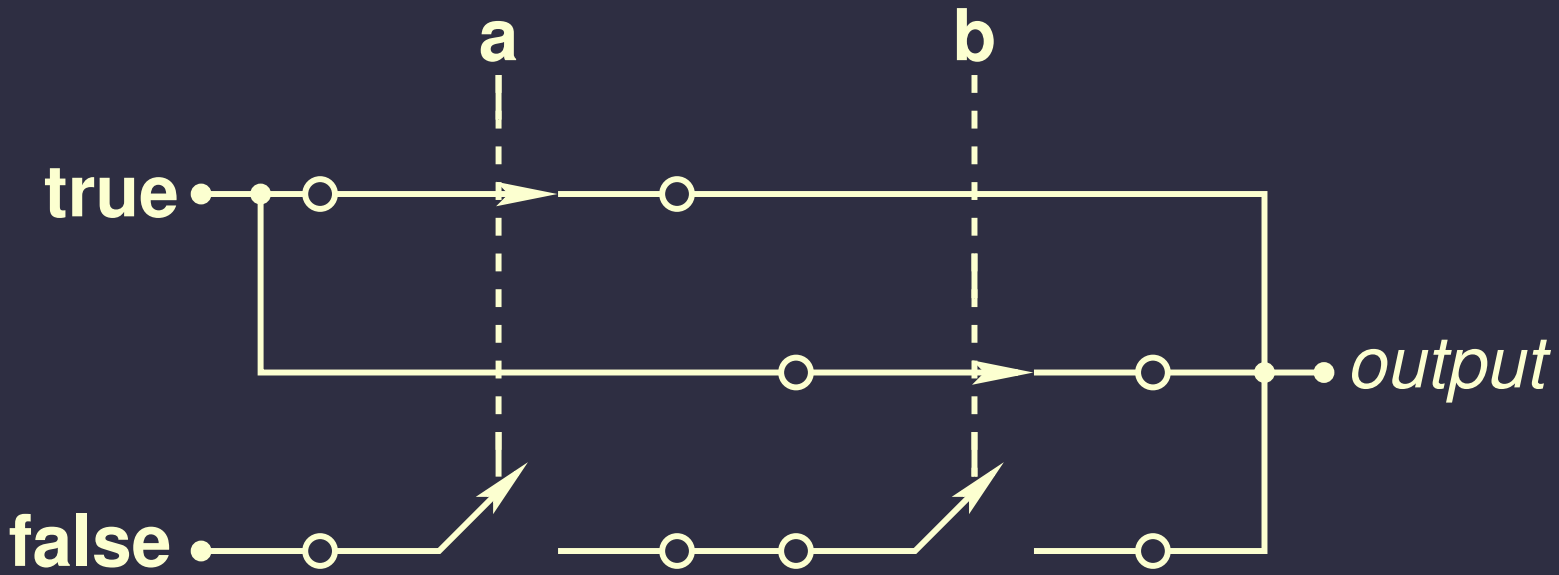
NAND gate

Therefore, *NAND* and *NOR* gates are used in CMOS design instead of *AND* and *OR* gates



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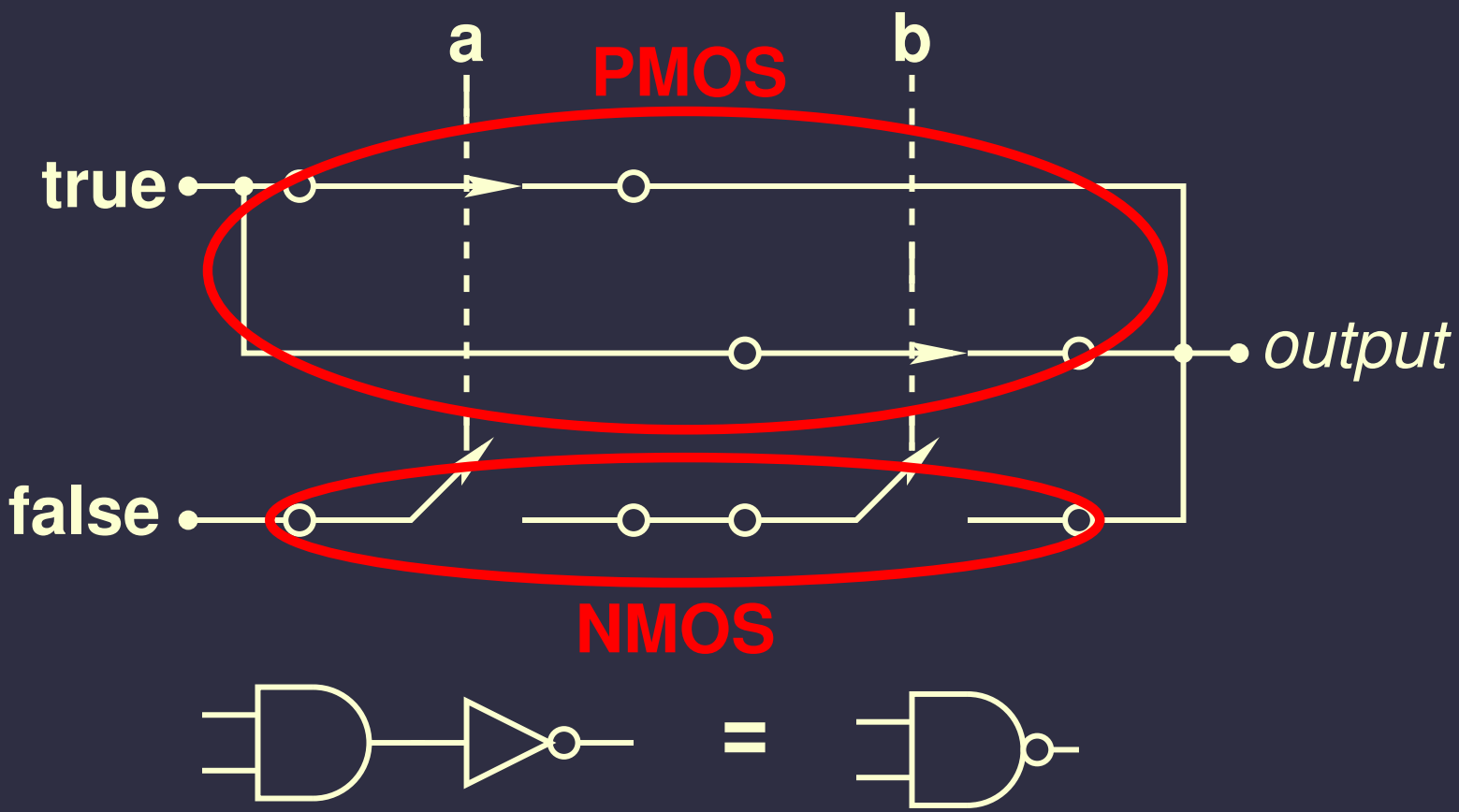
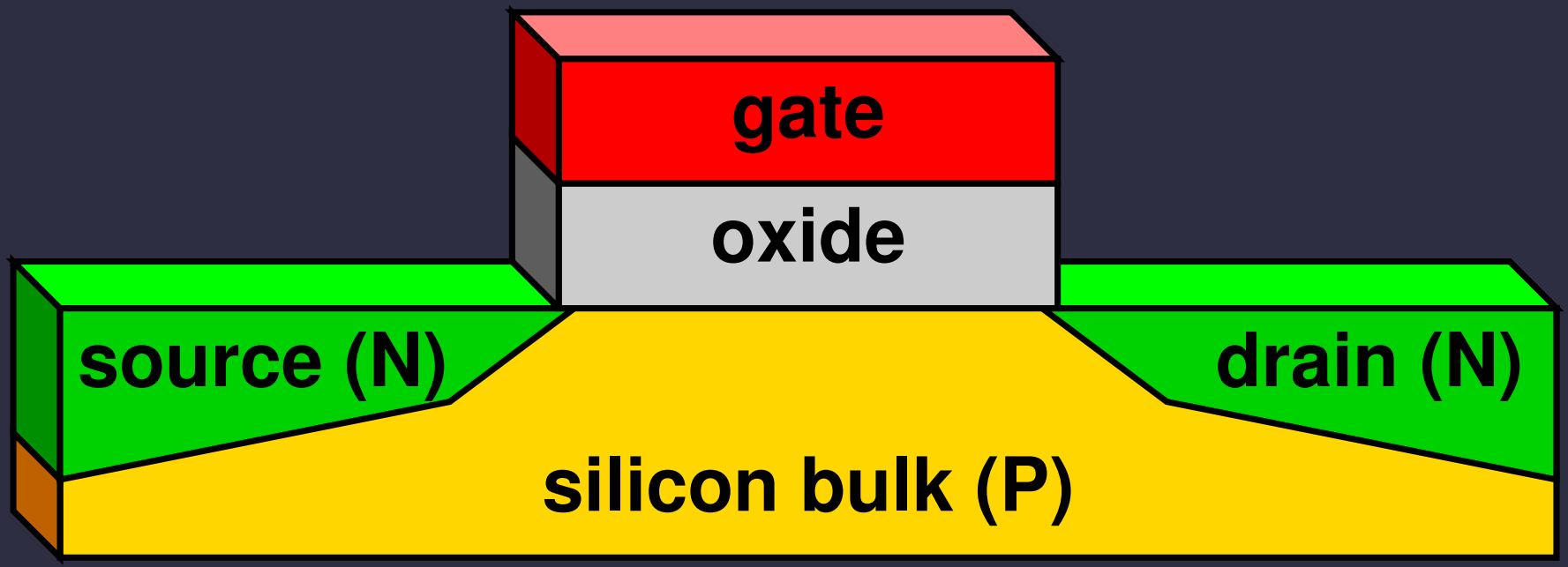


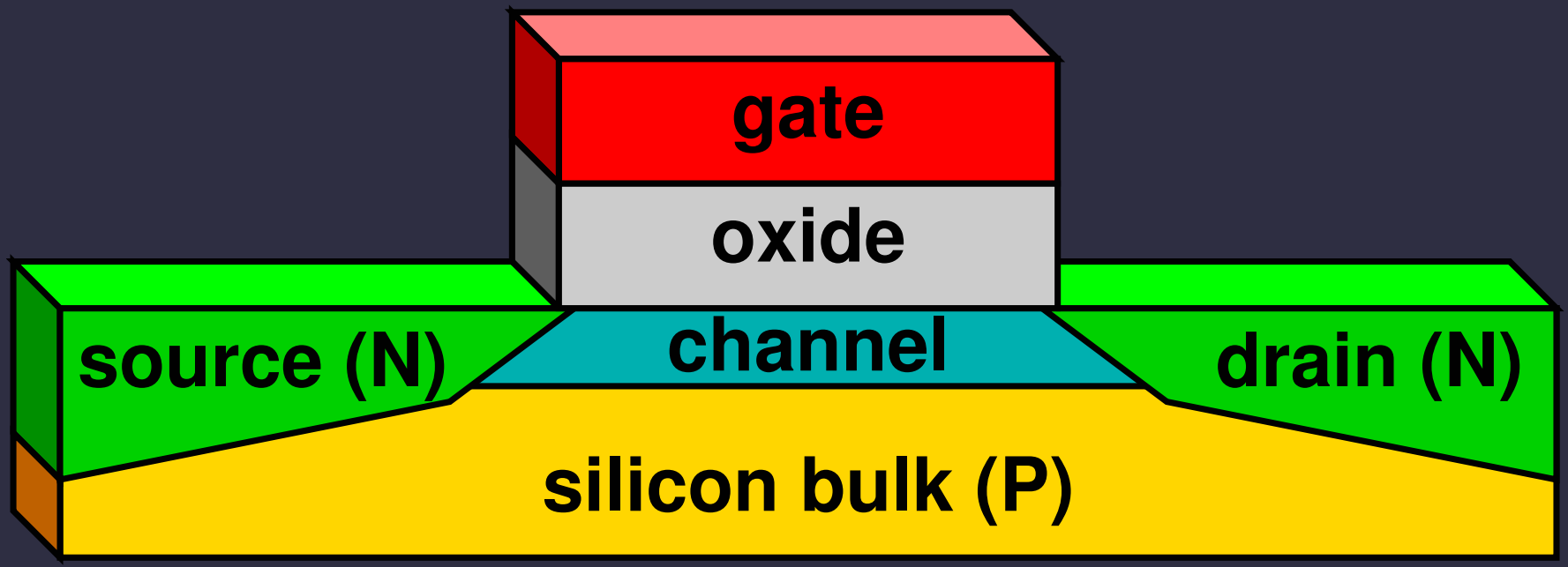
Transistors

- Basic device in NMOS and PMOS (CMOS) technologies
- Can be used to construct any logic gate

NMOS transistor



NMOS transistor



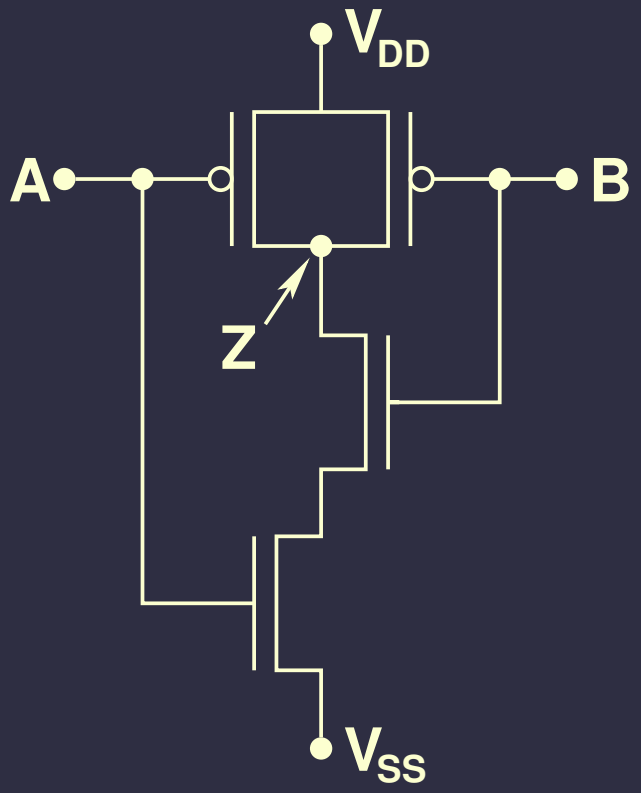
NMOS transistor

- Metal, oxide, semiconductor (MOS)
  - Then it was polysilicon, oxide, semiconductor
  - Now it is metal, hafnium-based low-k dielectric, semiconductor
- P-type bulk silicon doped with positively charged ions
- N-type diffusion regions doped with negatively charged ions
- Gate can be used to pull a few electrons near the oxide
  - Forms channel region, conduction from source to drain starts

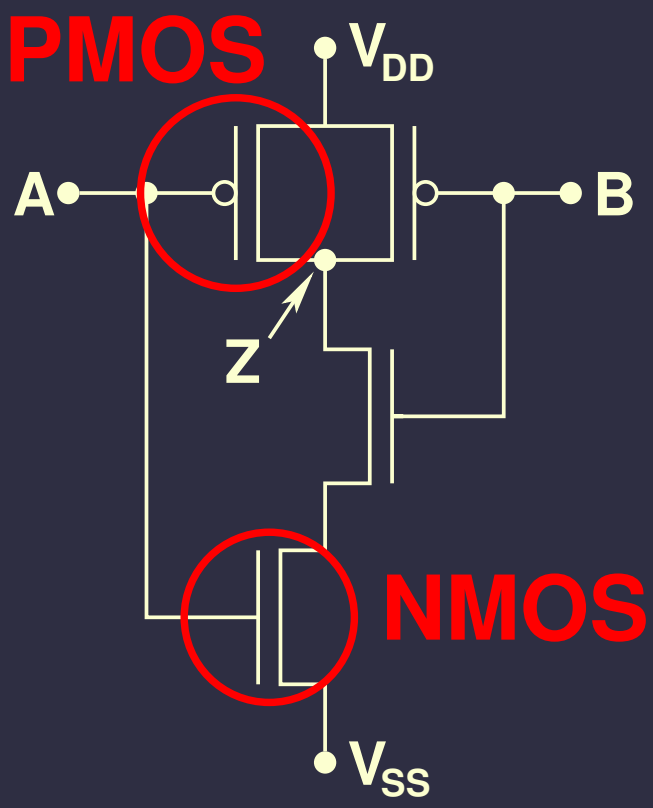
CMOS

- NMOS turns on when the gate is high
- PMOS just like NMOS, with N and P regions swapped
- PMOS turns on when the gate is low
- NMOS good at conducting low (0s)
- PMOS good at conducting high (1s)
- Use NMOS and PMOS transistors together to build circuits
  - Complementary metal oxide silicon (CMOS)

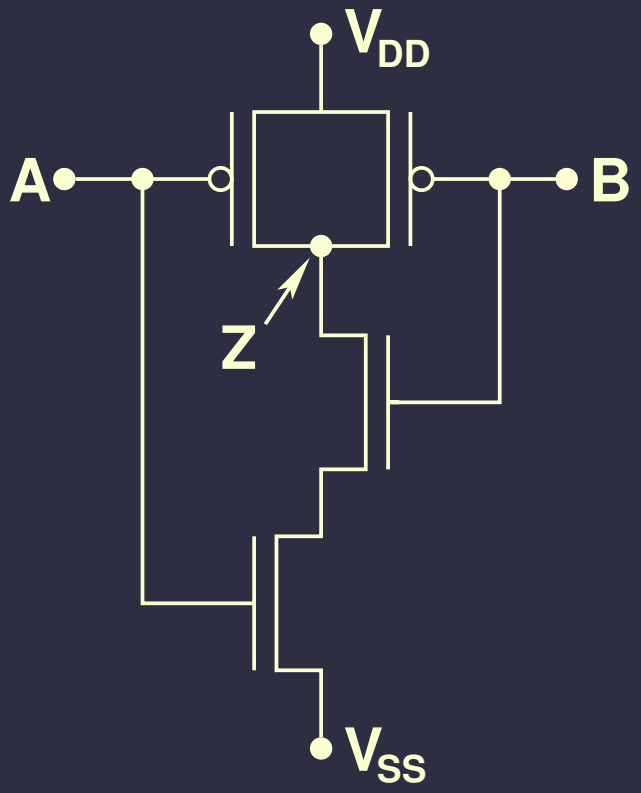
CMOS NAND gate



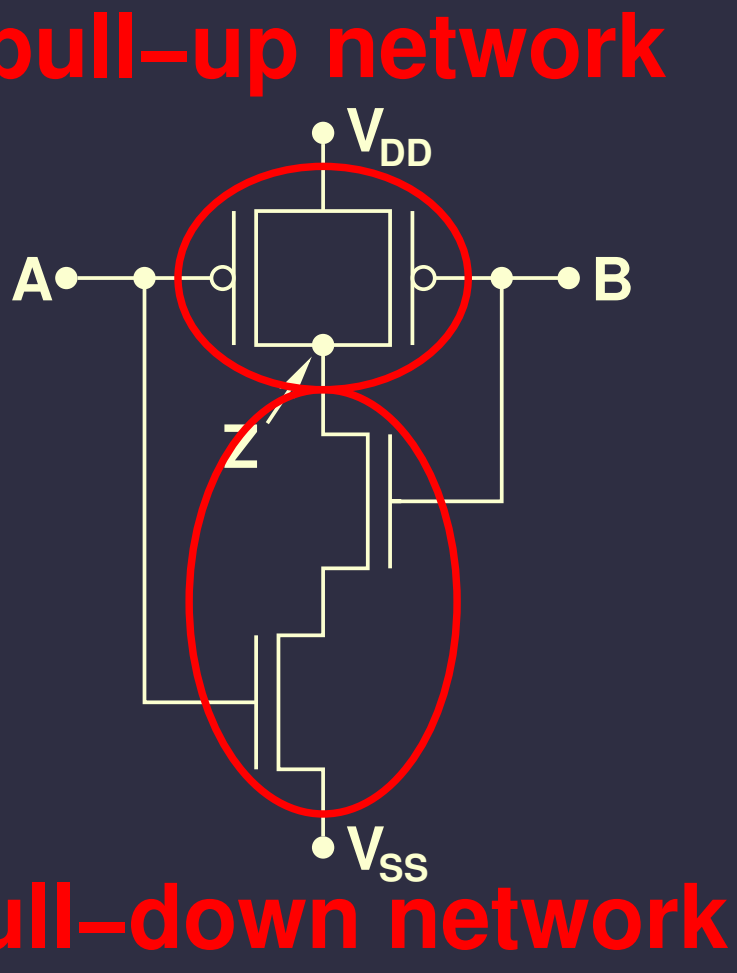
CMOS NAND gate



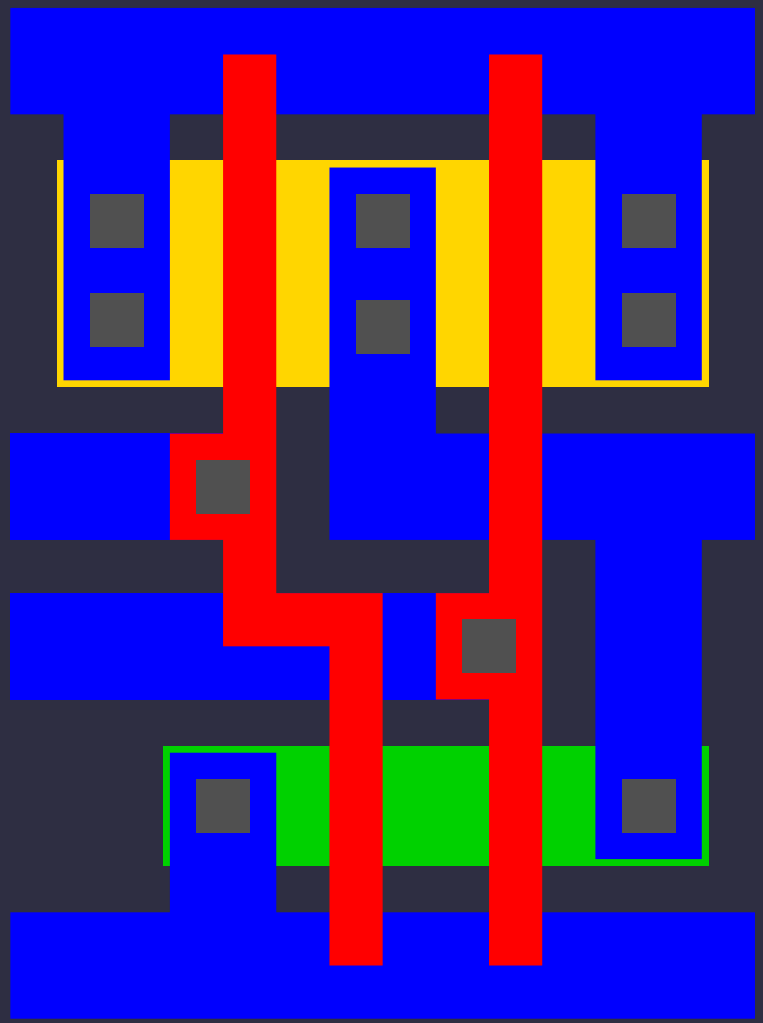
CMOS NAND gate



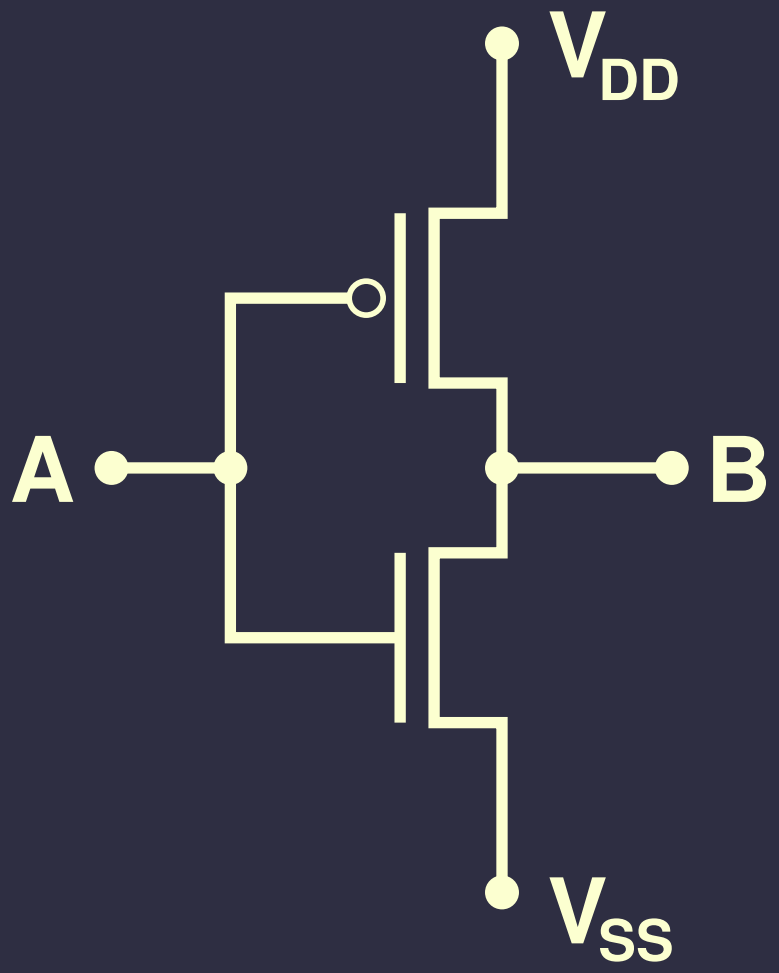
CMOS NAND gate



CMOS NAND gate layout



CMOS inverter operation



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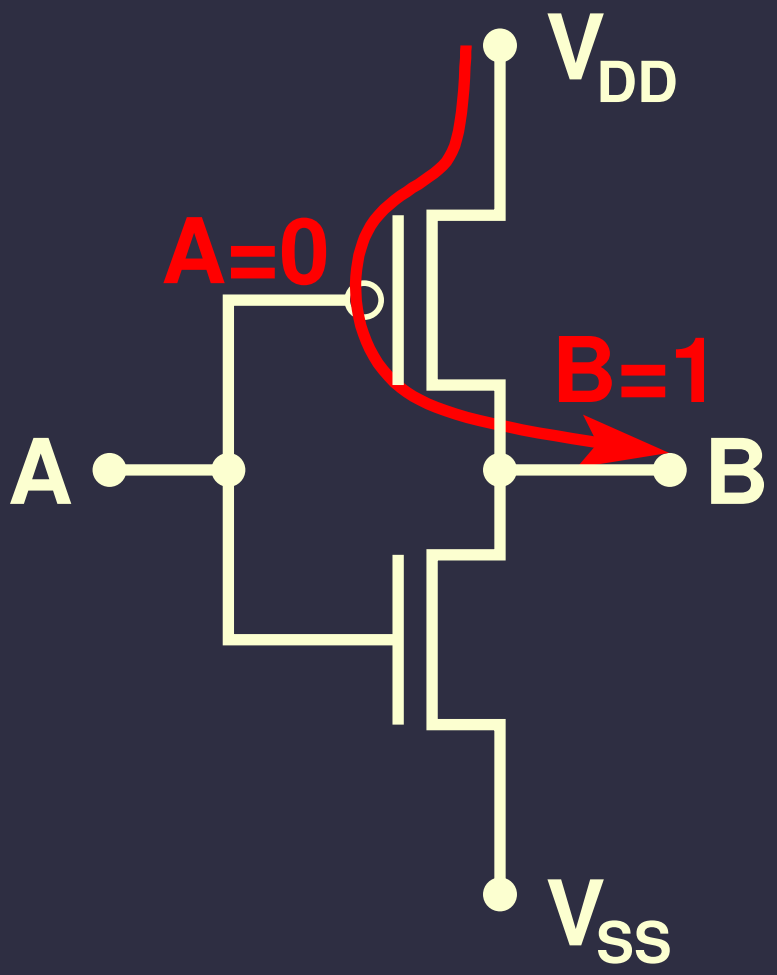

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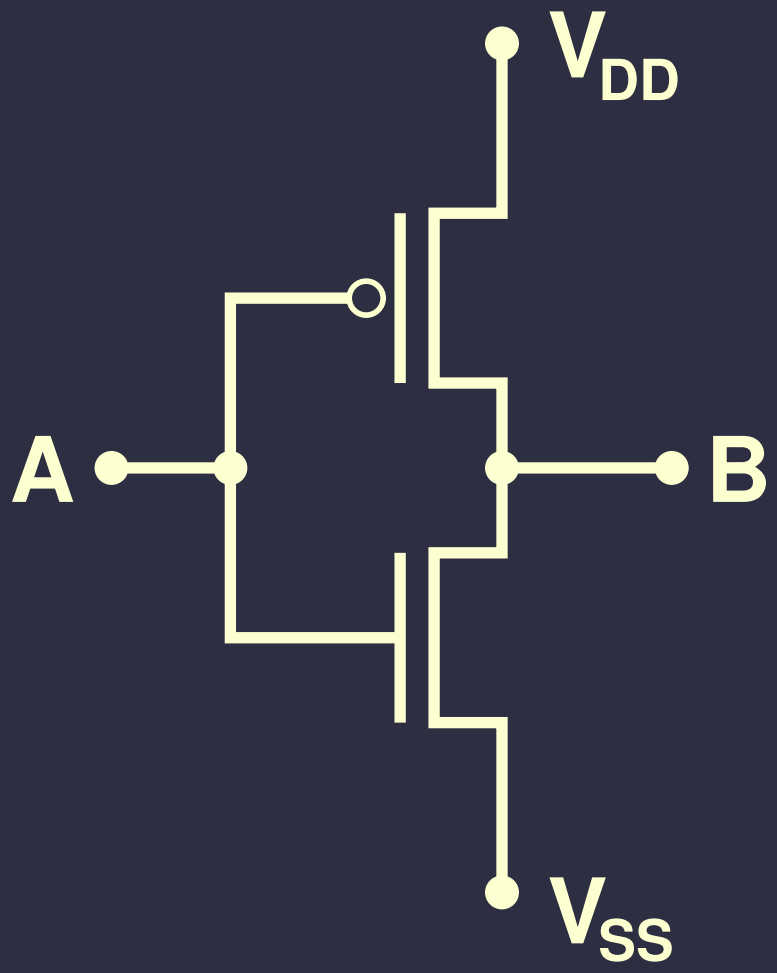

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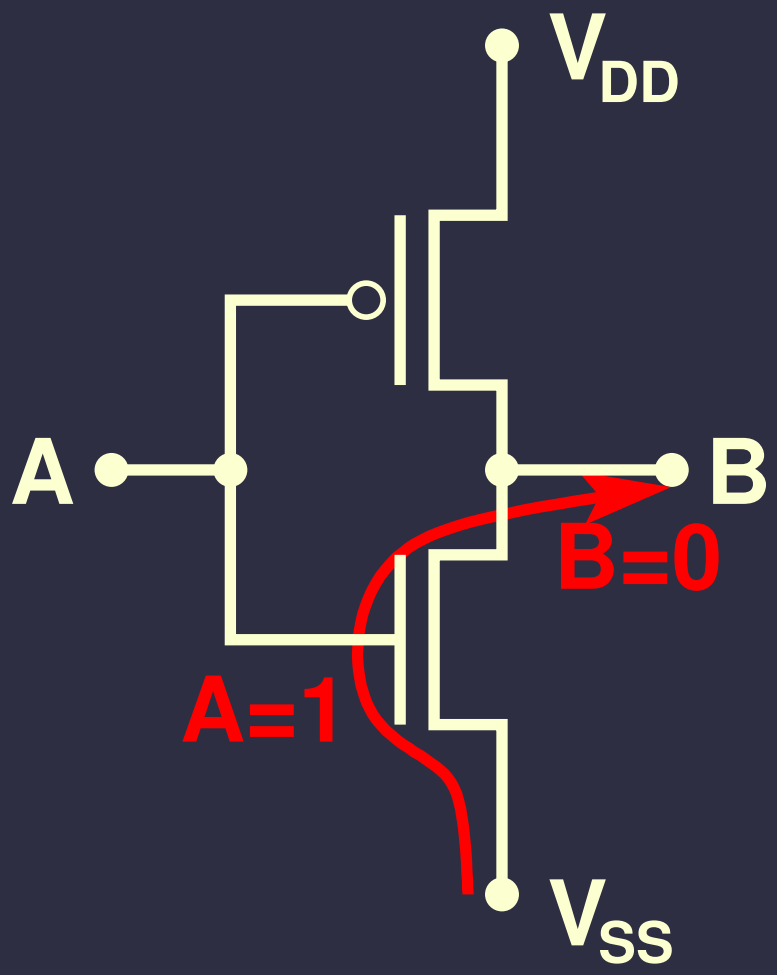

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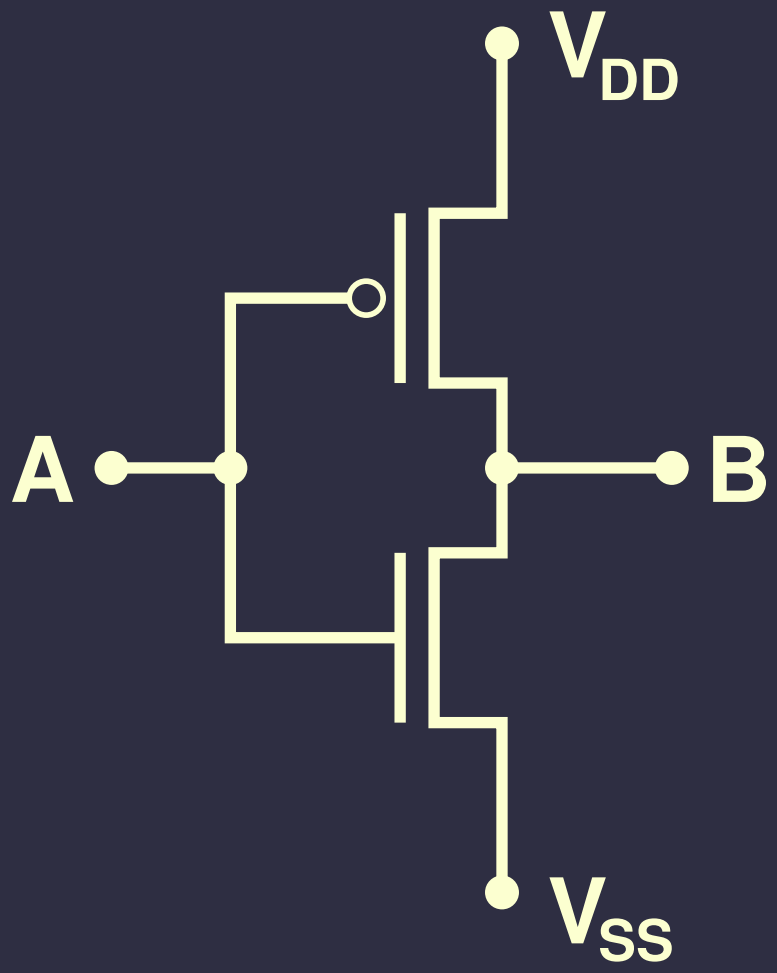

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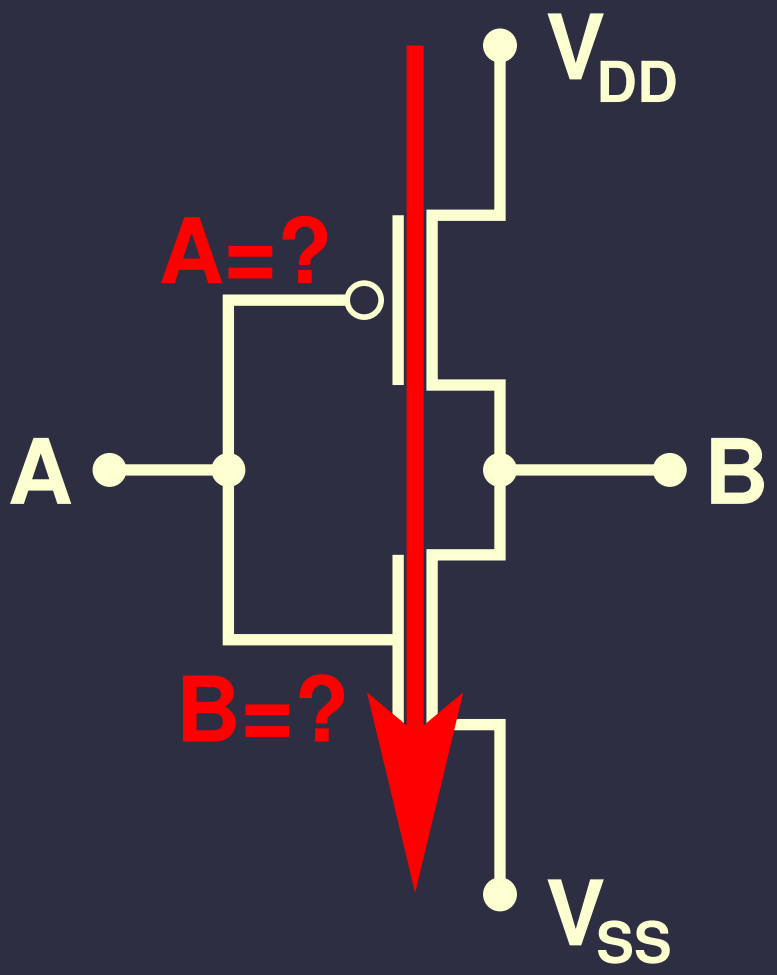
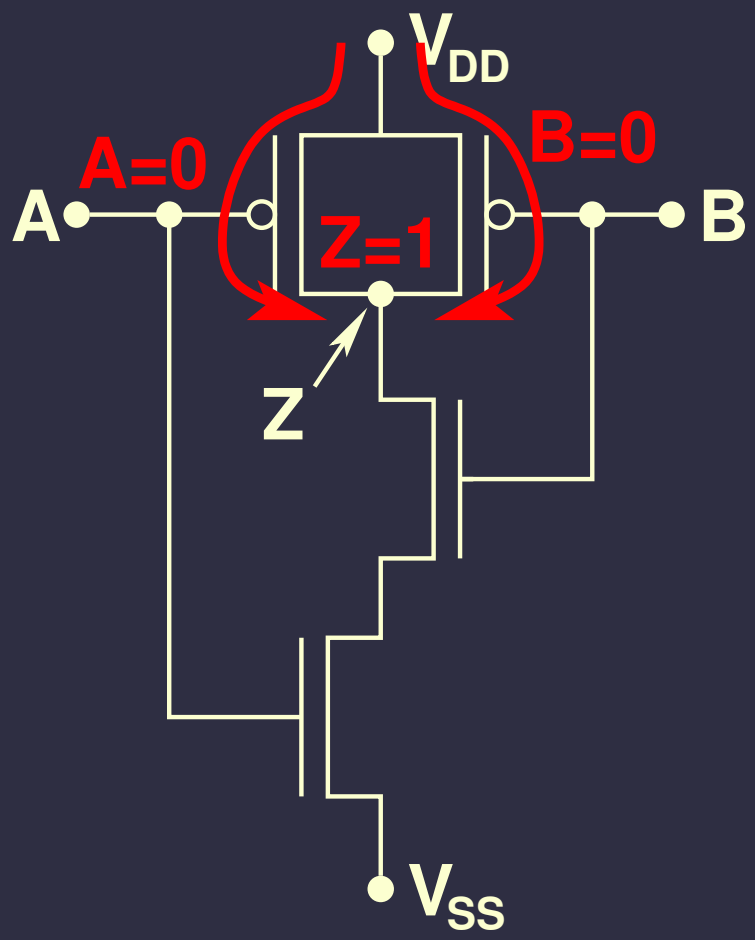
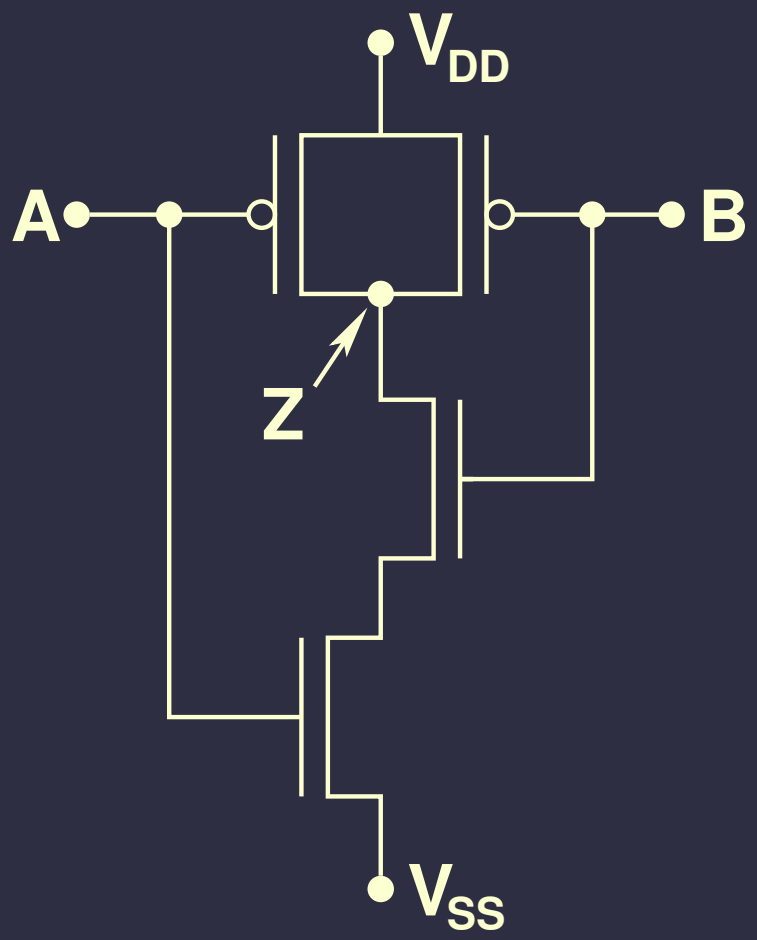


NAND operation



NAND operation



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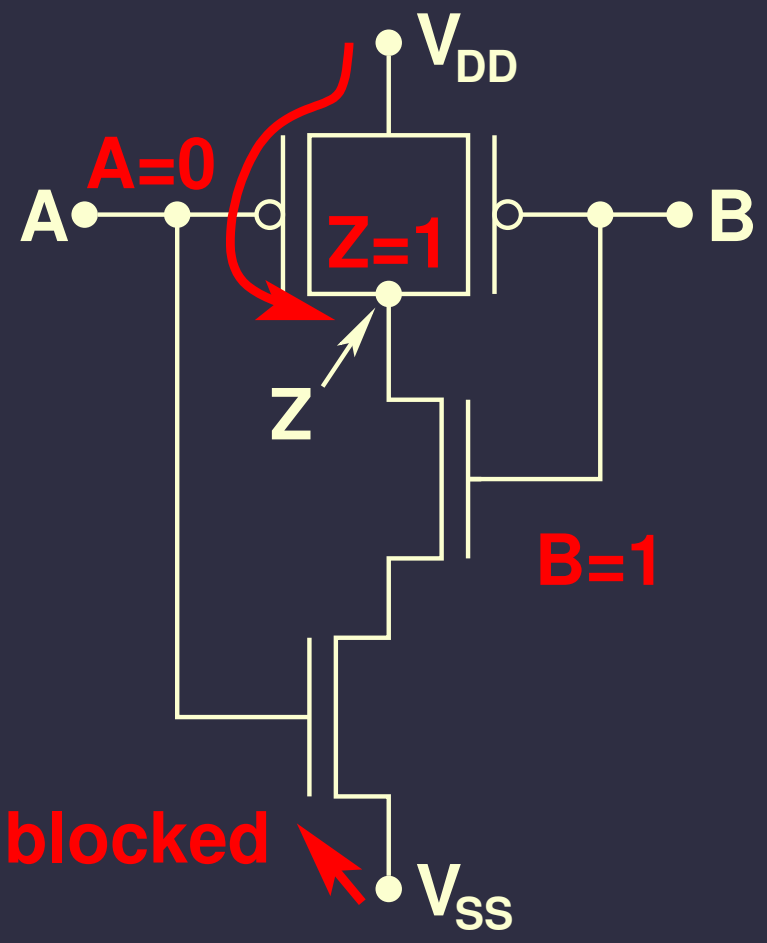

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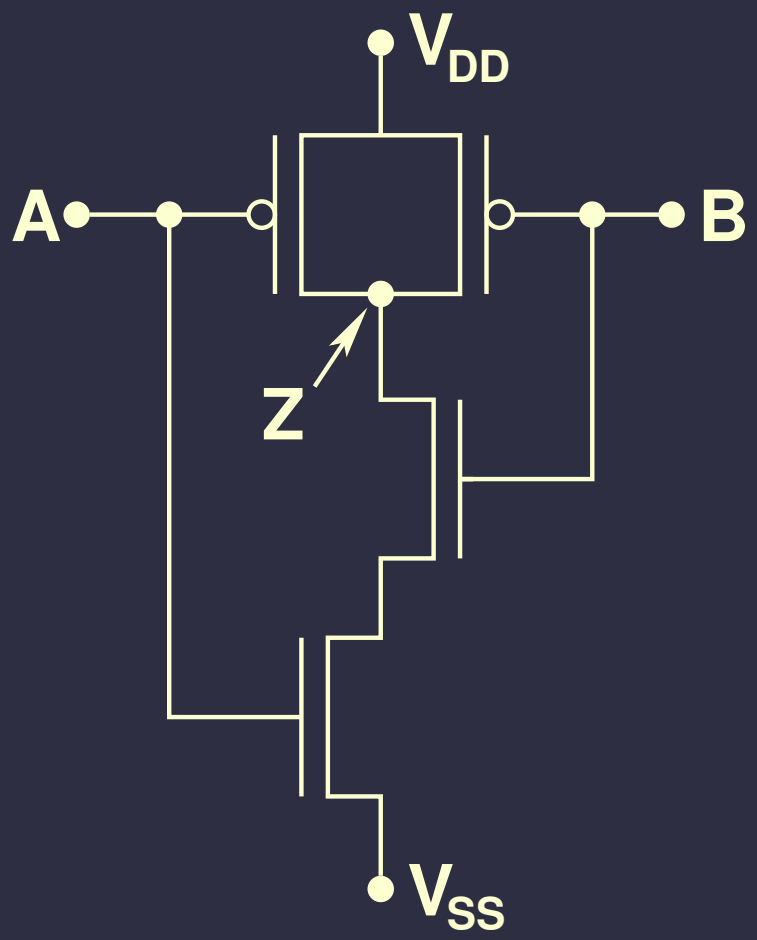

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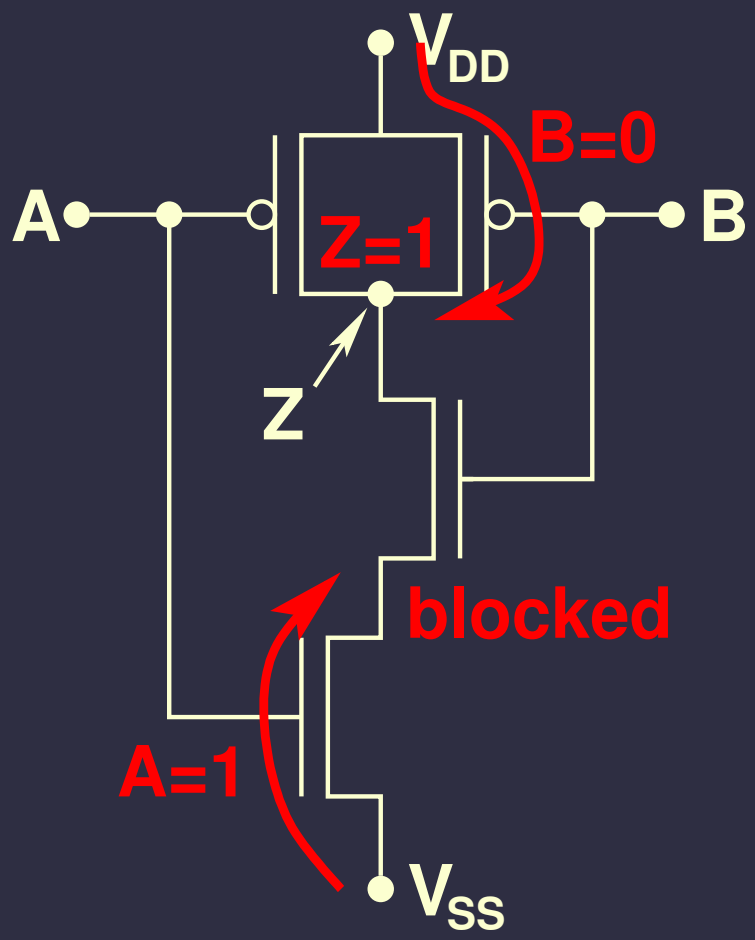

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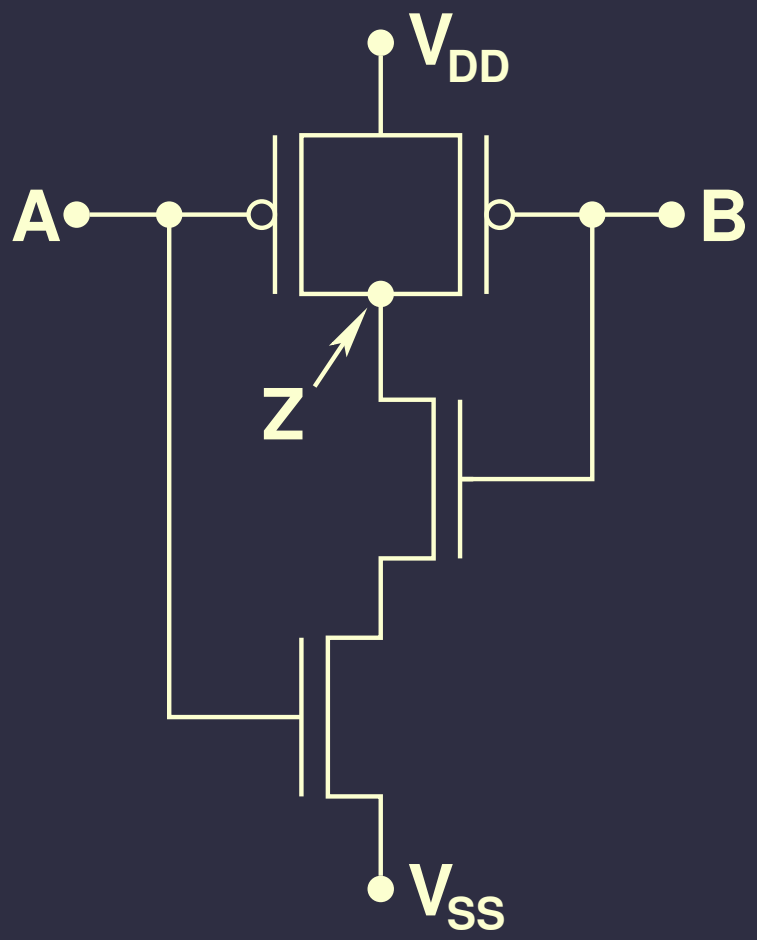

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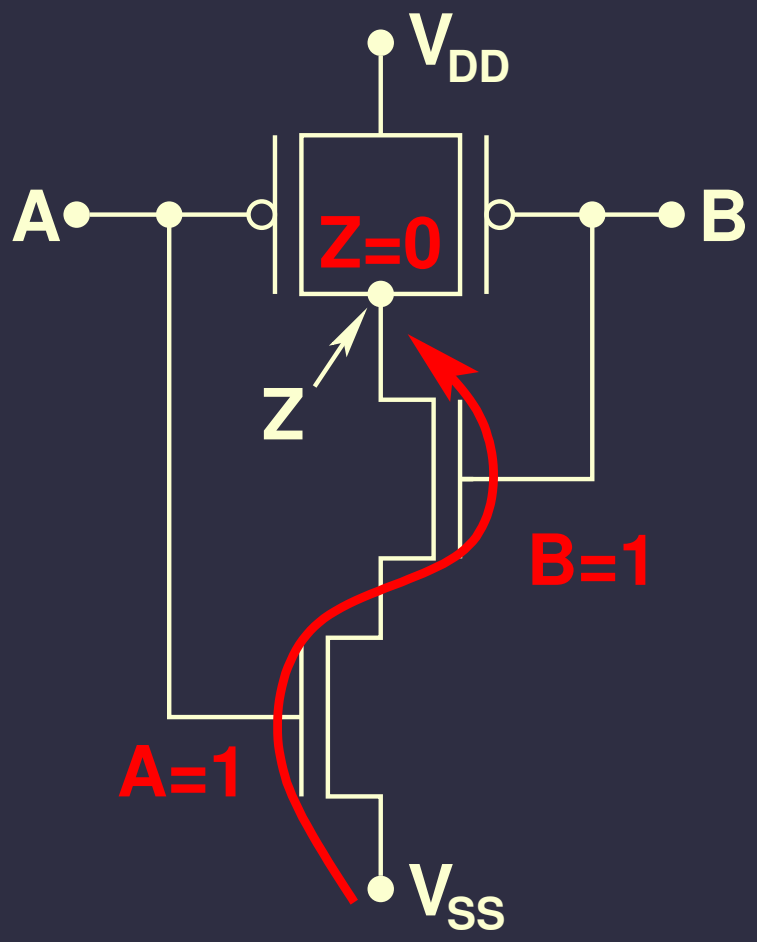

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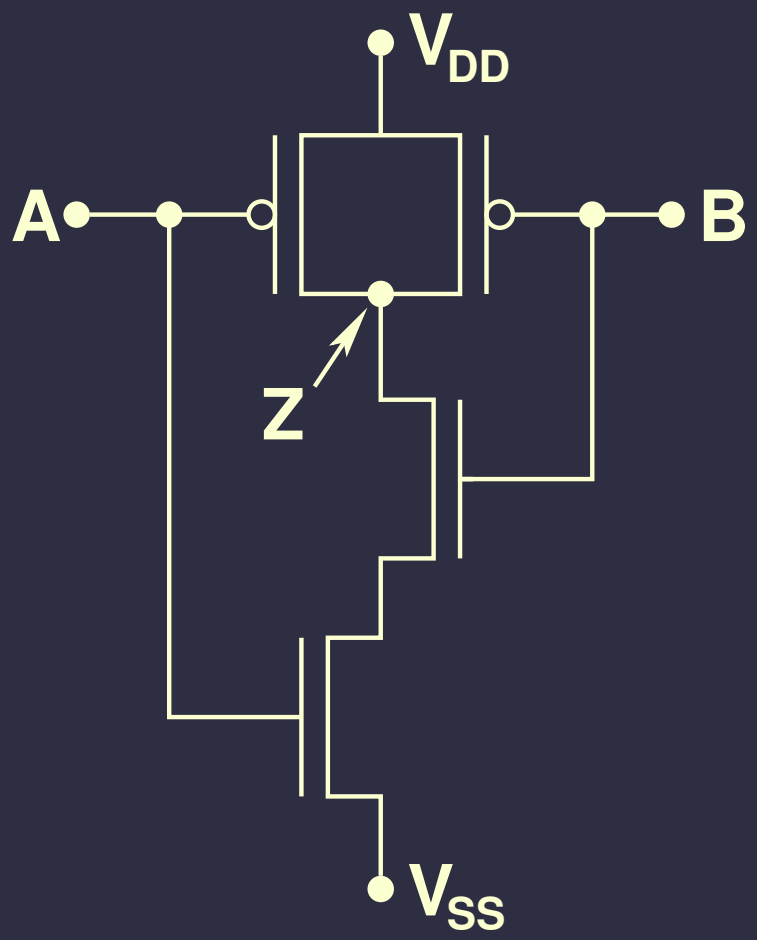
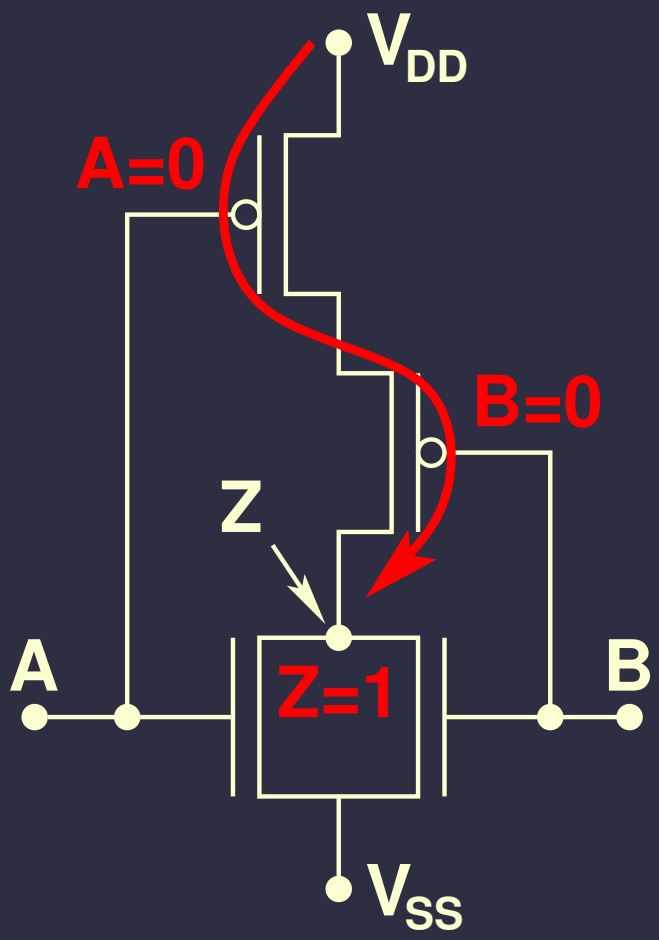
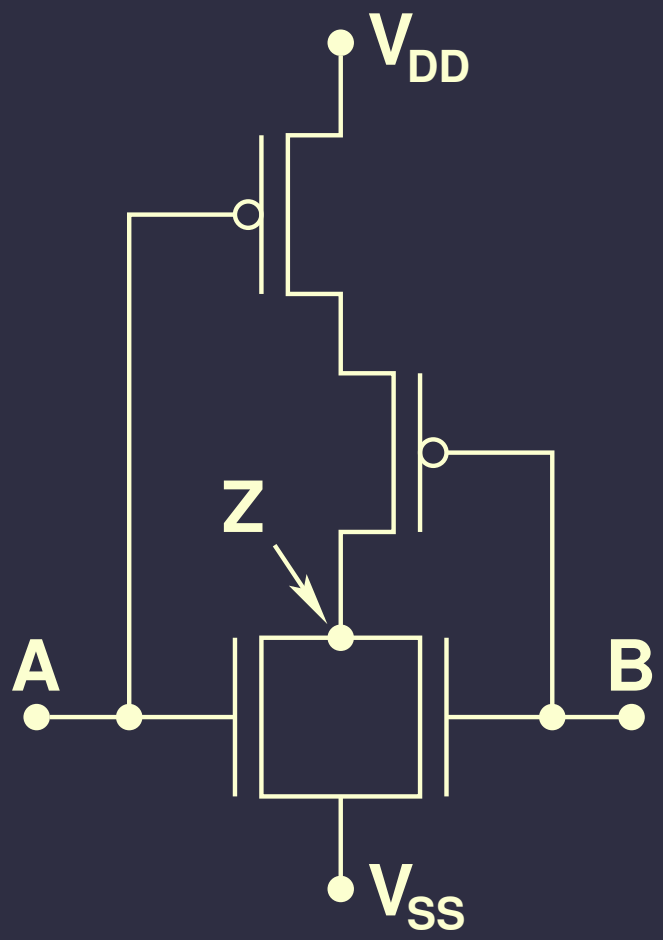


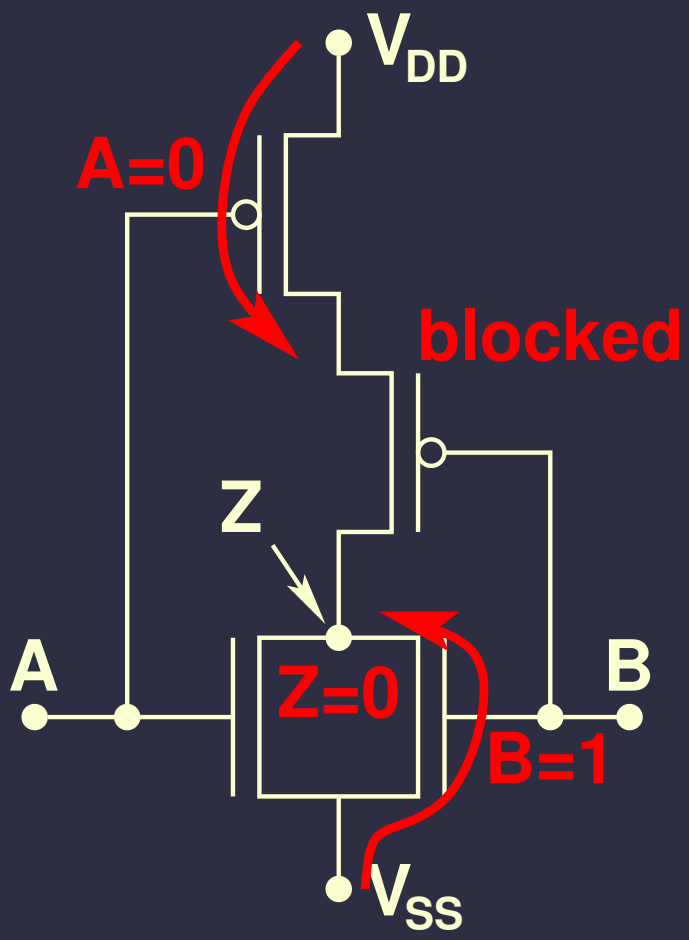
NOR operation



NOR operation



NOR operation



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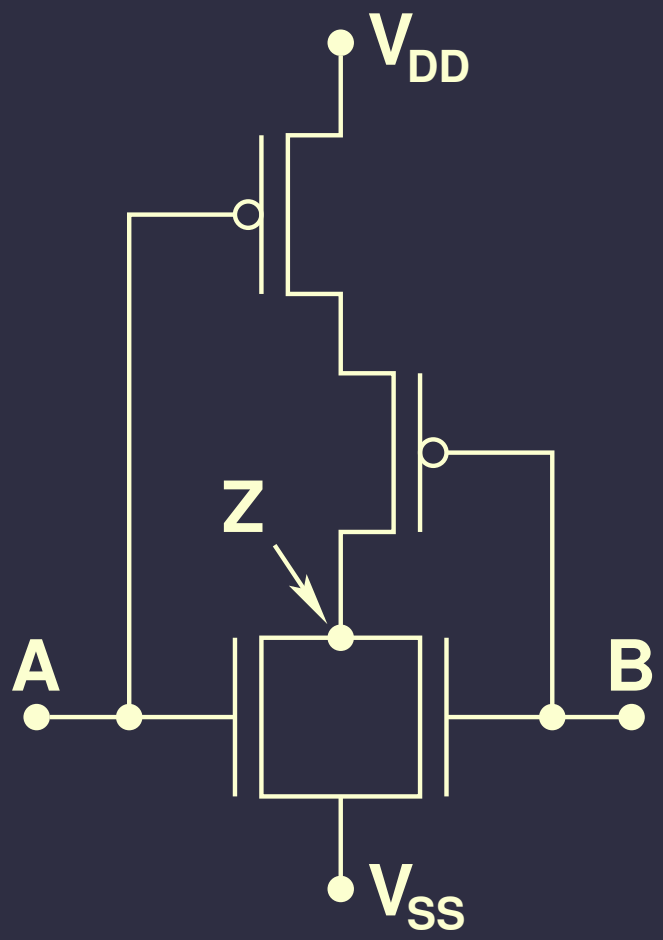

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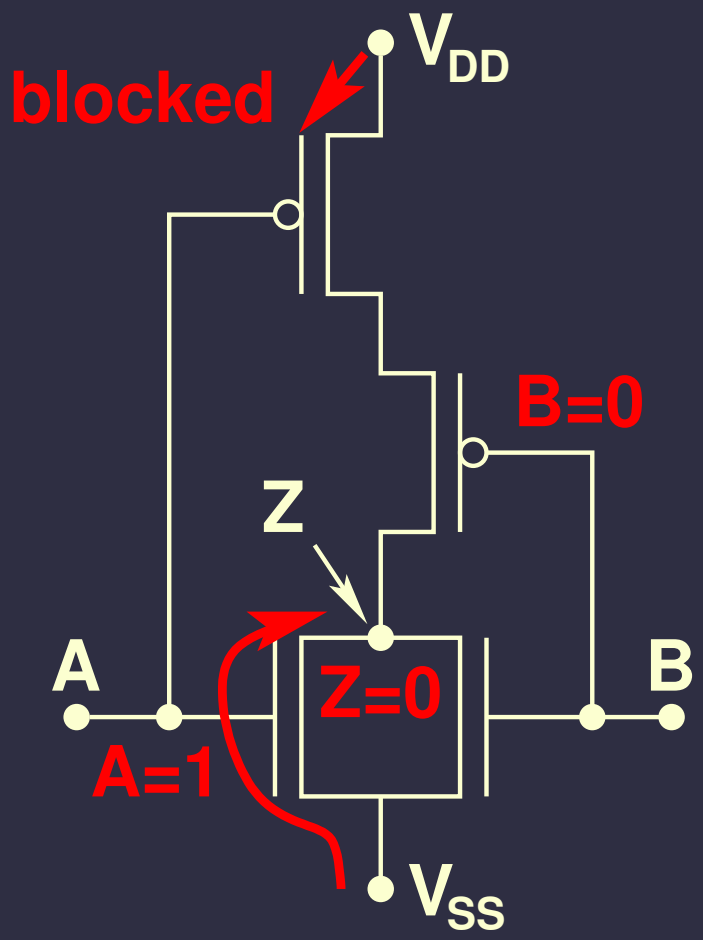

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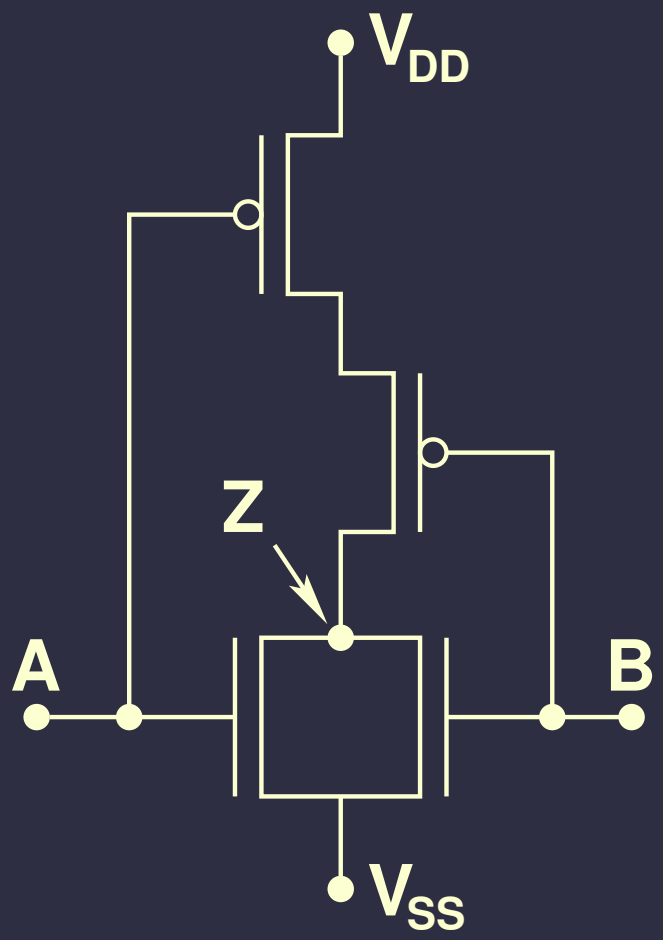

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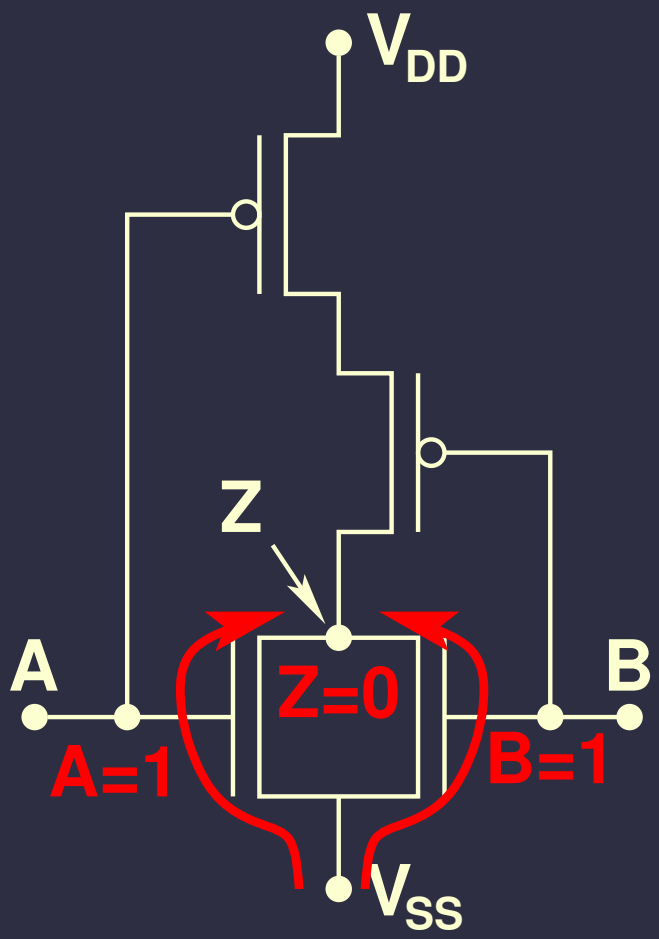

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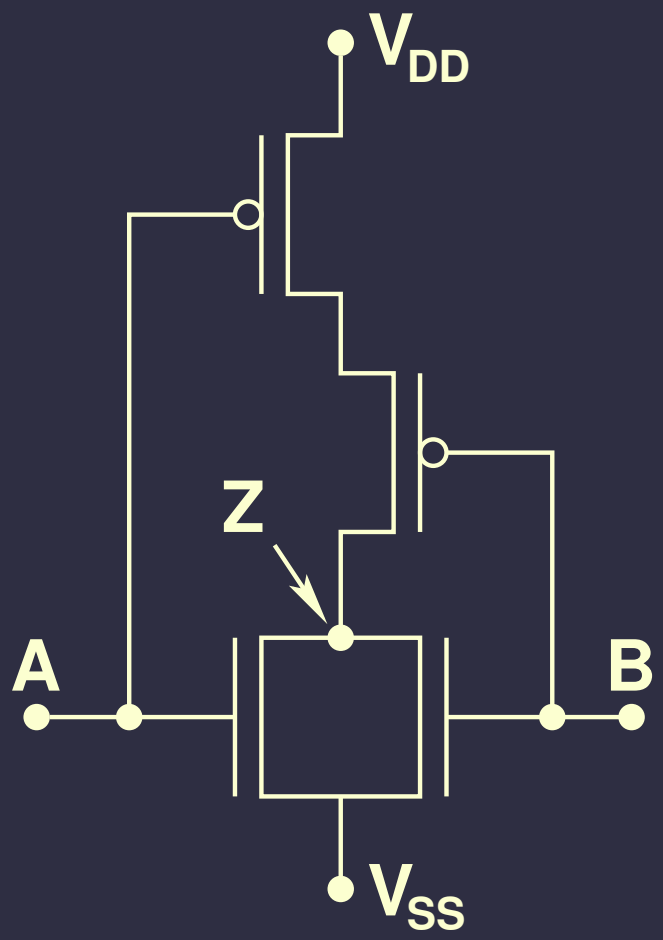

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1. Transistors in digital systems
   - Switches
   - CMOS
   - AND and OR are harder than NAND and NOR
   - Transmission gates

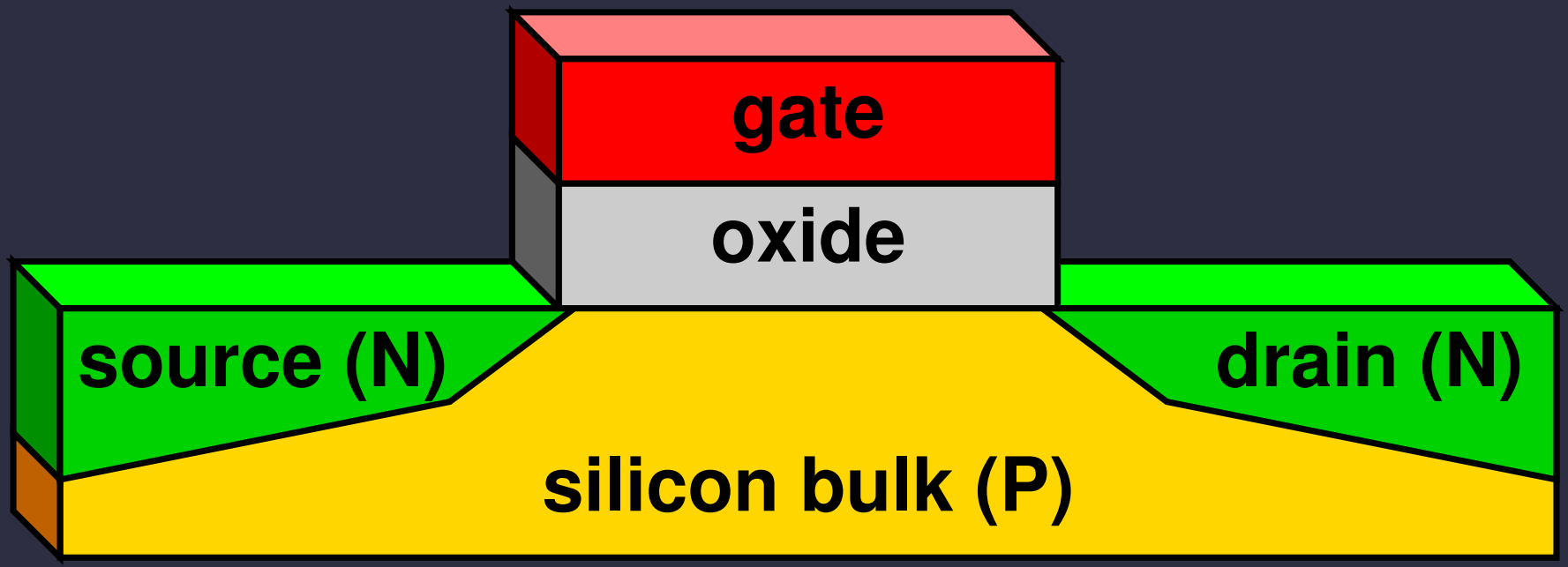
Non-ideality of NMOS/PMOS transistors

- Recall that NMOS transmits low values easily...
- ...transmits high values poorly
- PMOS transmits high values easily...
- ...transmits low values poorly

Non-ideality of NMOS/PMOS transistors

- V_T, or threshold voltage, is commonly 0.7 V
- NMOS conducts when $V_{GS} > V_T$
- PMOS conducts when $V_{GS} < -V_T$
- What happens if an NMOS transistor's source is high?
- Or a PMOS transistor's source is low?
- Alternatively, if one states that $V_{TN} = 0.7$ V and $V_{TP} = -0.7$ V then NMOS conducts when $V_{GS} > V_{TN}$ and PMOS conducts when $V_{GS} < V_{TP}$

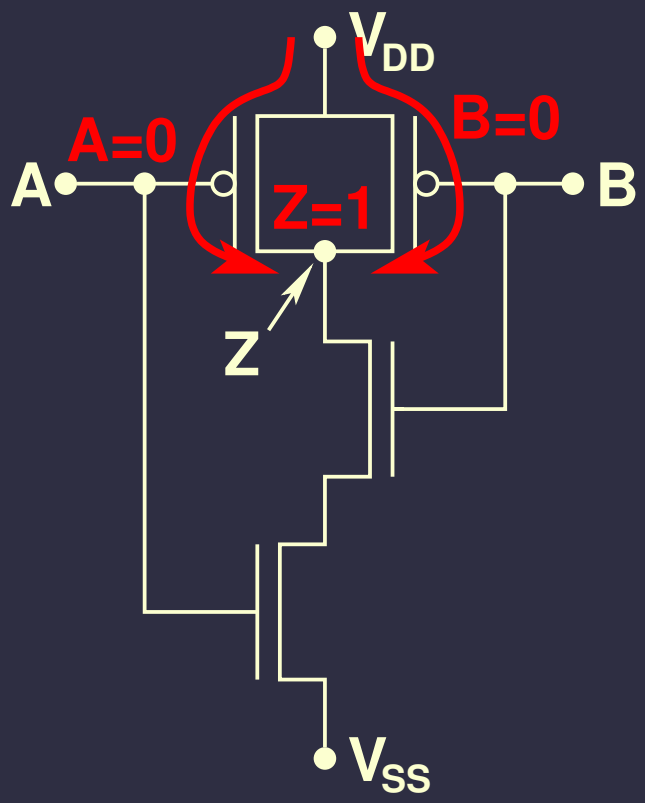
NMOS transistor



Non-ideality of NMOS/PMOS transistors

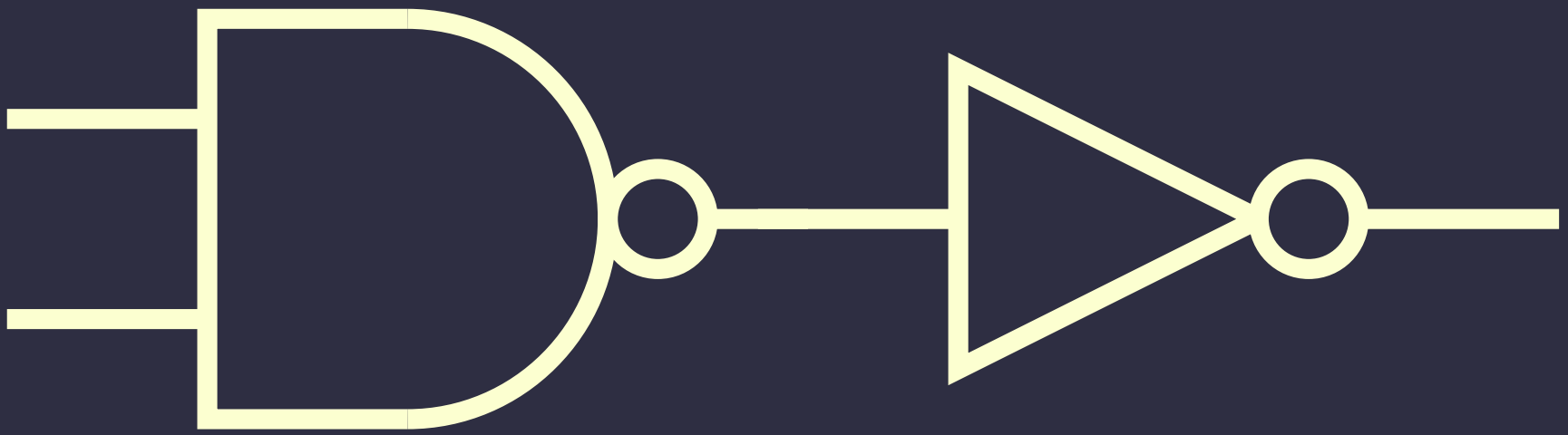
- If an NMOS transistor's input were V_{DD} (high), for $V_{GS} > V_{TN}$, the gate would require a higher voltage than V_{DD}
- If an PMOS transistor's input were V_{SS} (low), for $V_{GS} < V_{TP}$, the gate would require a lower voltage than V_{SS}

Implications of non-ideality



NAND/NOR easy to build in CMOS

Implications of non-ideality



AND/OR requires more area, power, time

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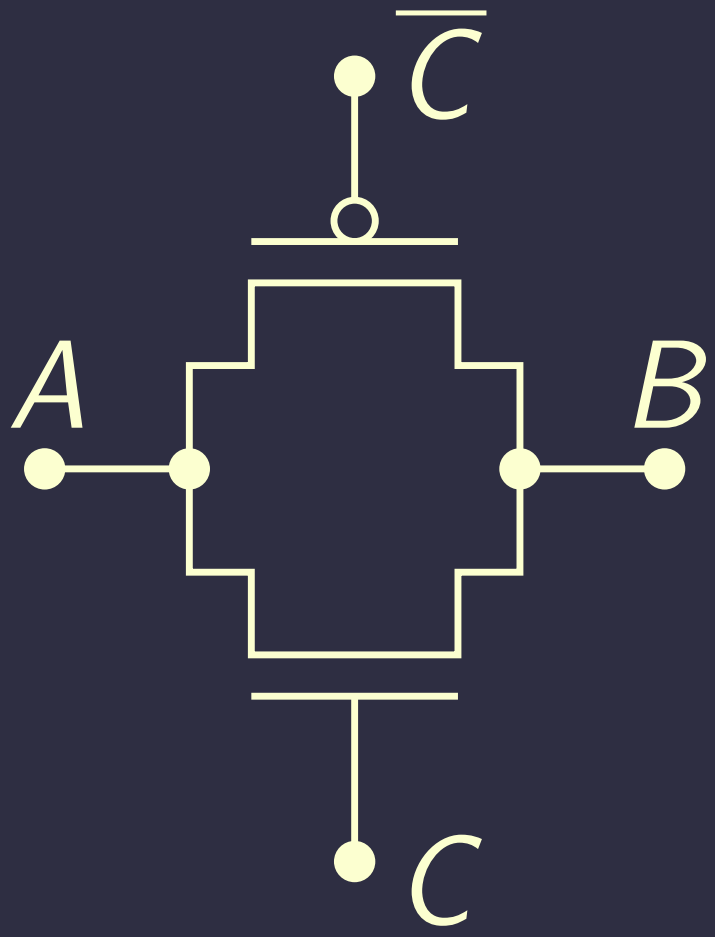
CMOS transmission gates (switches)

- NMOS is good at transmitting 0s
  - Bad at transmitting 1s
- PMOS is good at transmitting 1s
  - Bad at transmitting 0s
- To build a switch, use both: CMOS

CMOS transmission gate (TG)



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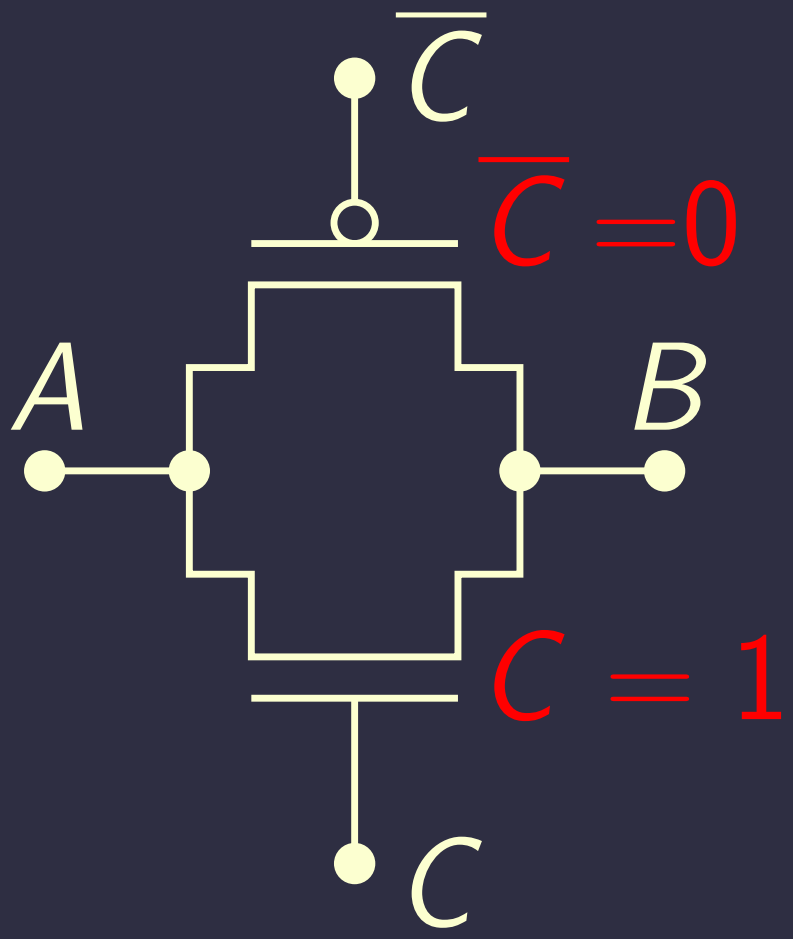

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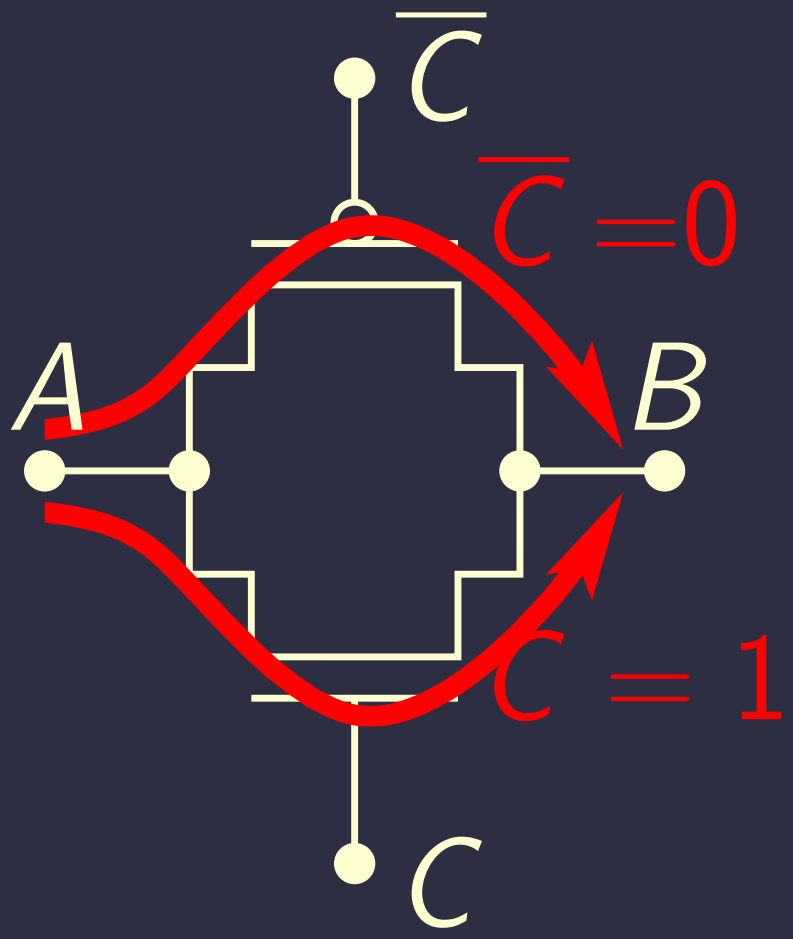

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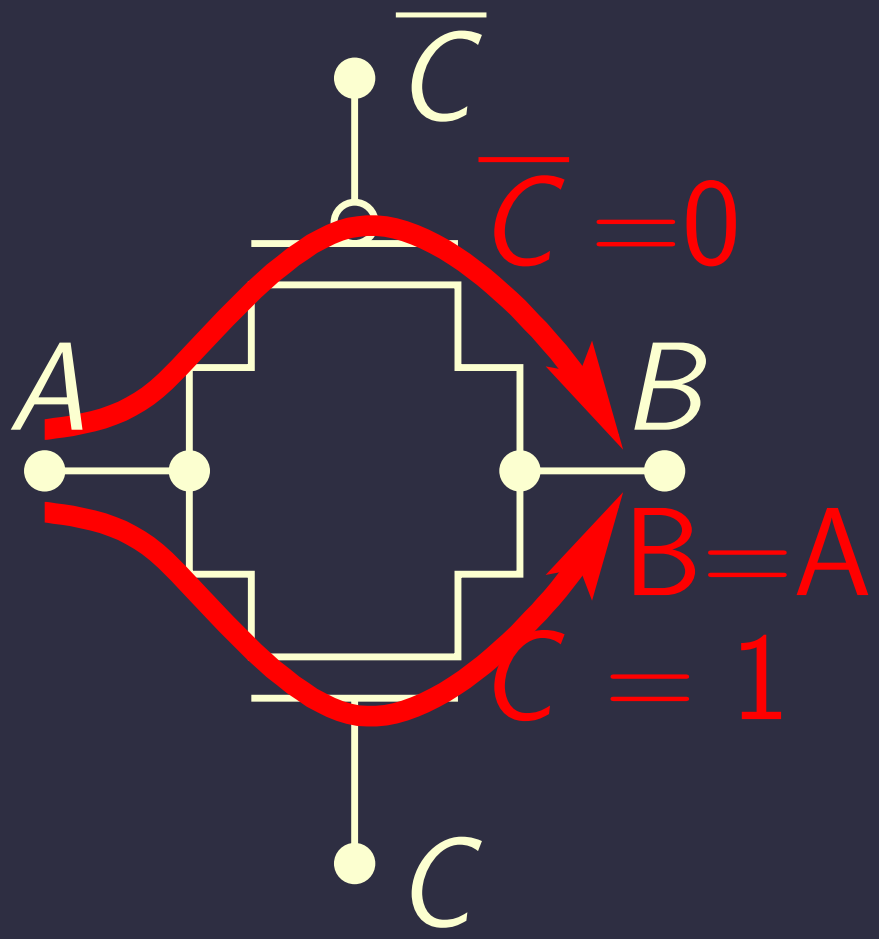

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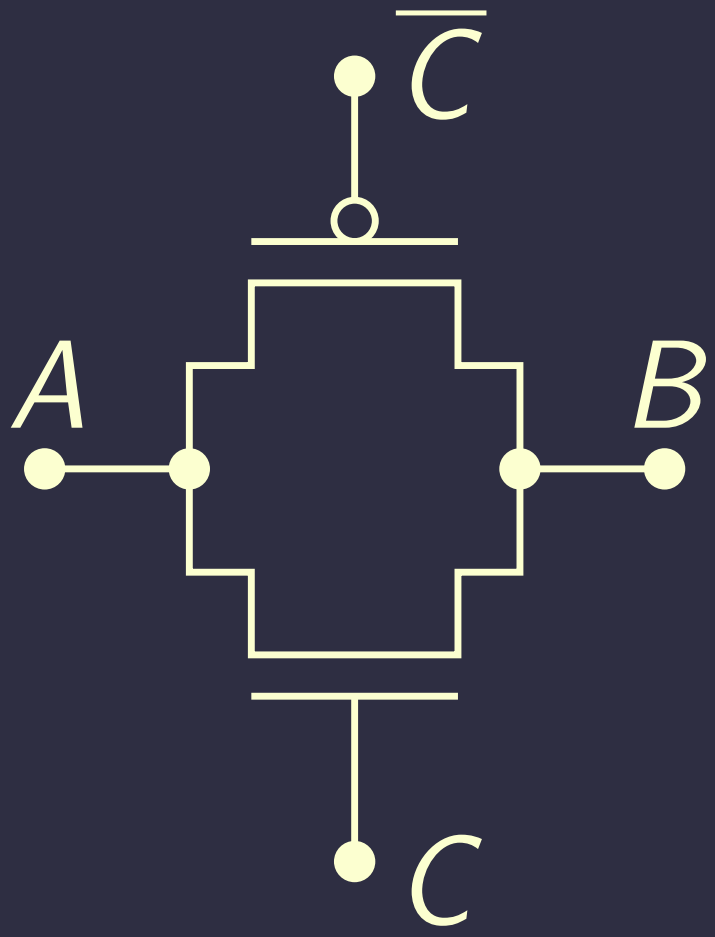

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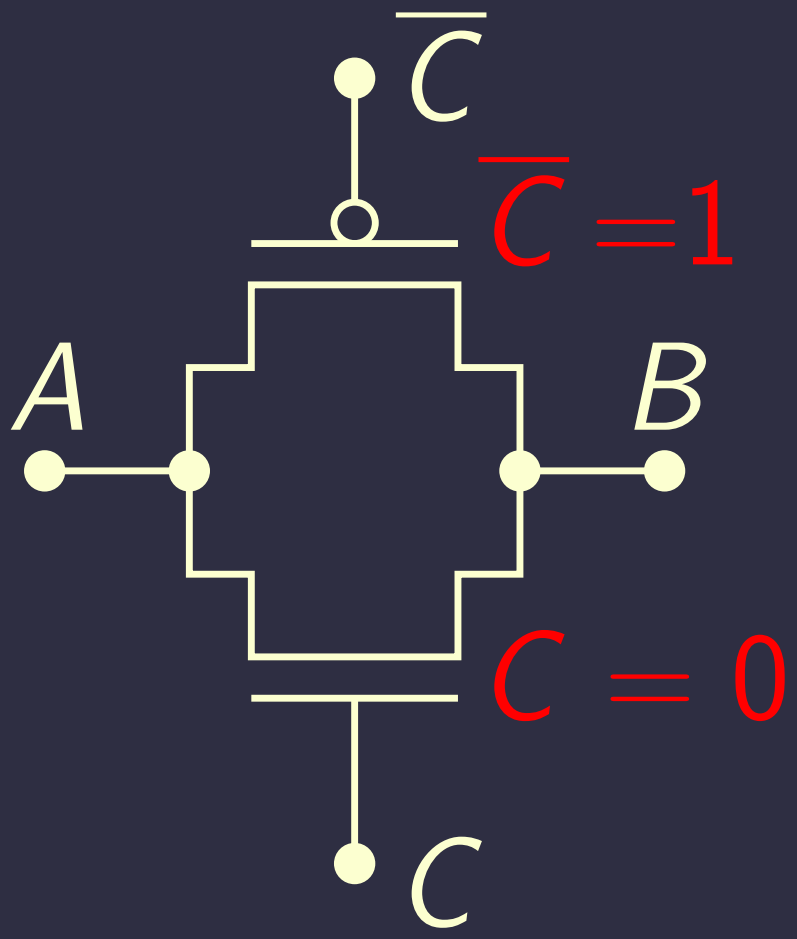

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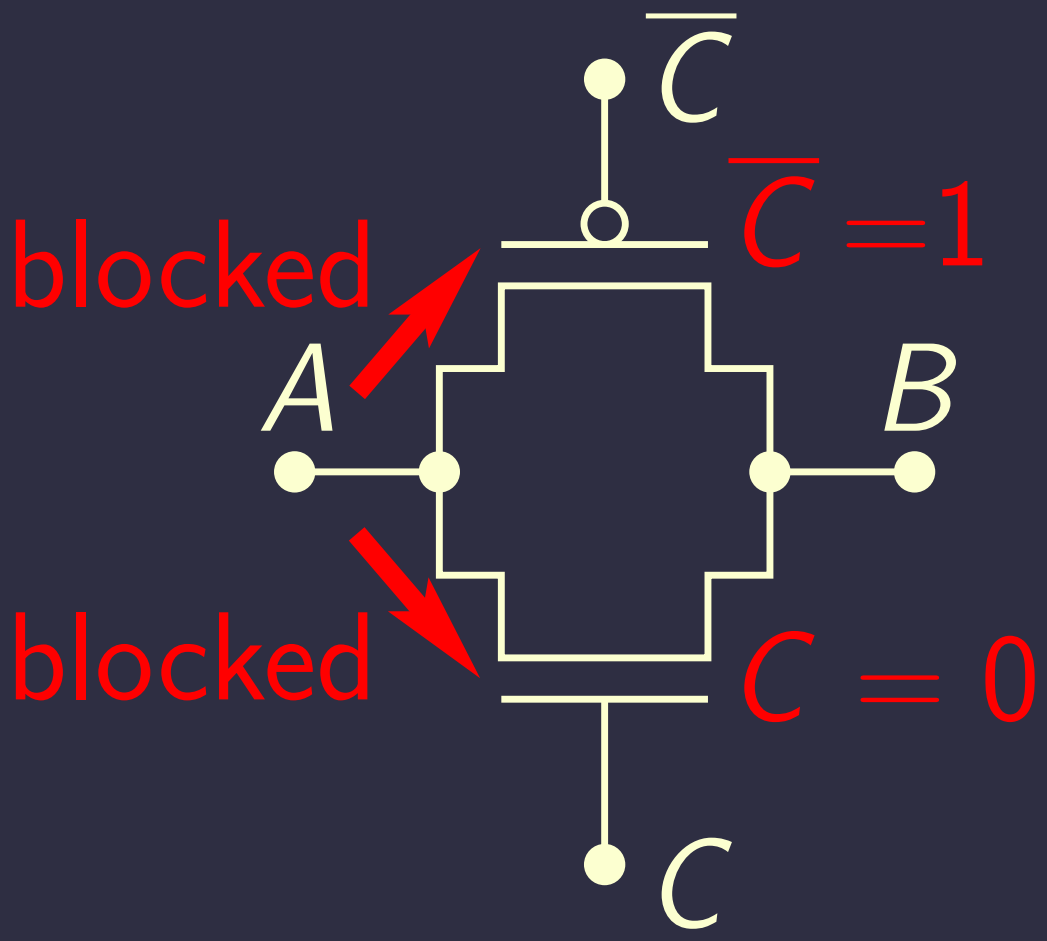

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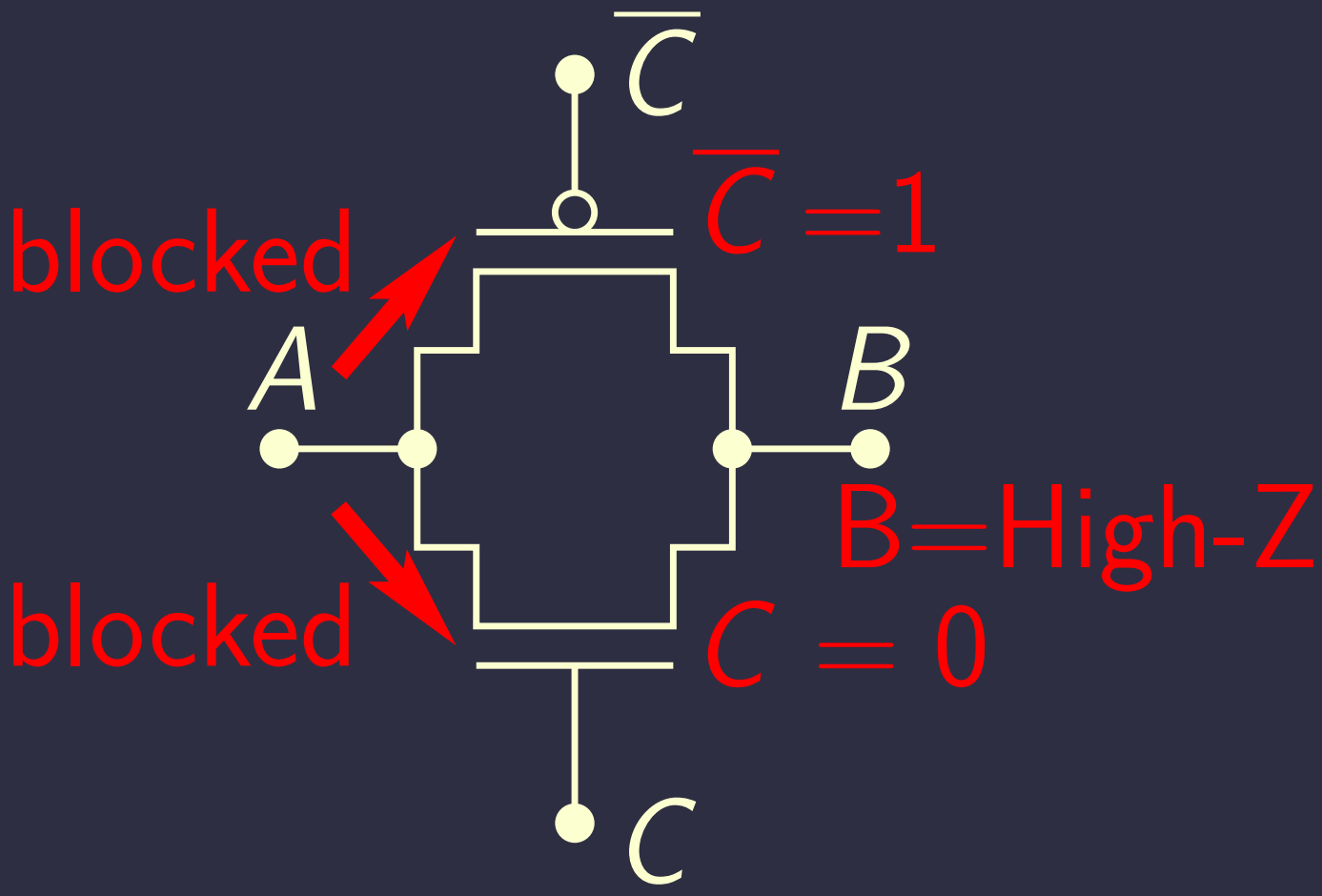

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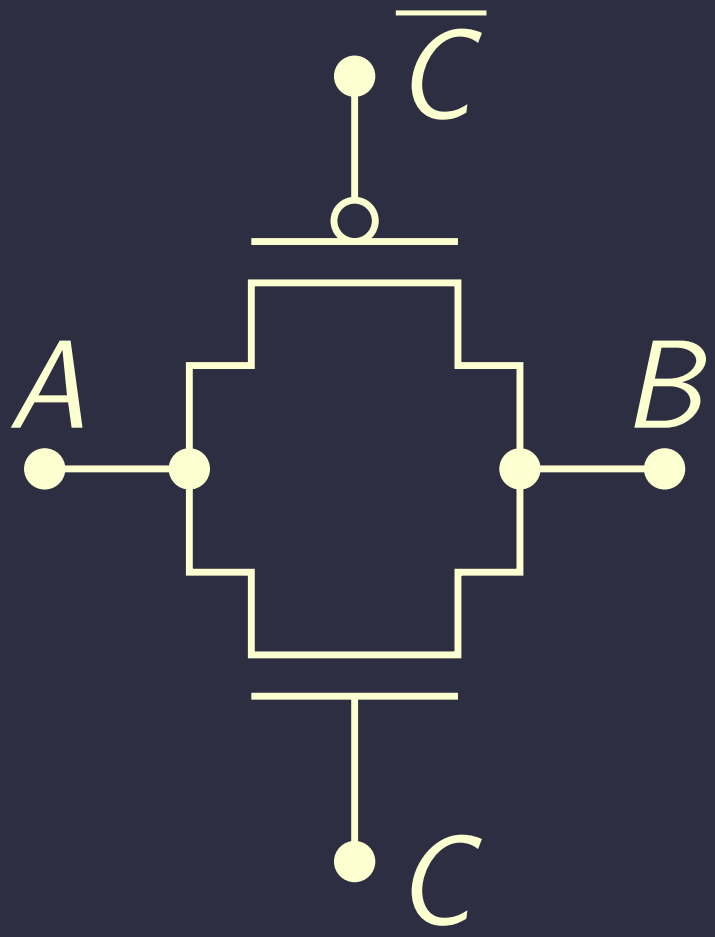
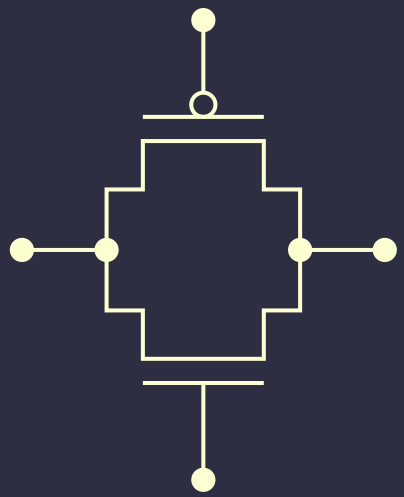
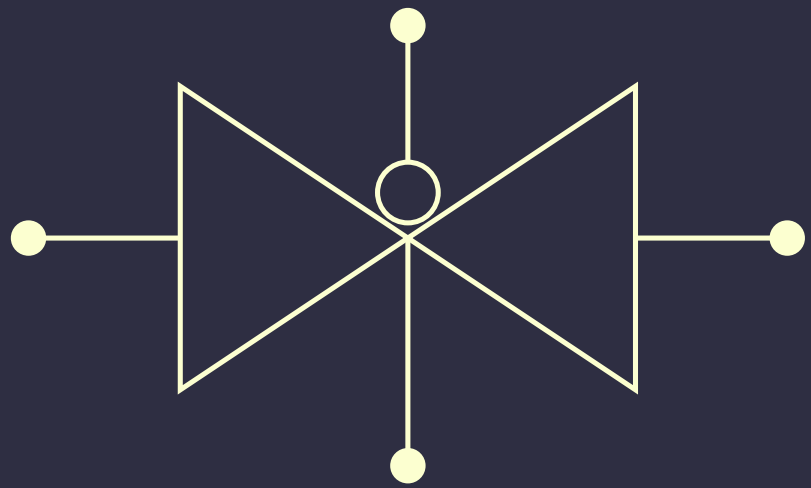


Other TG diagram





What can we build with TGs?

- Anything. . . try some examples.

Outline

Section outline

2. Homework
   - Assignment
   - Details

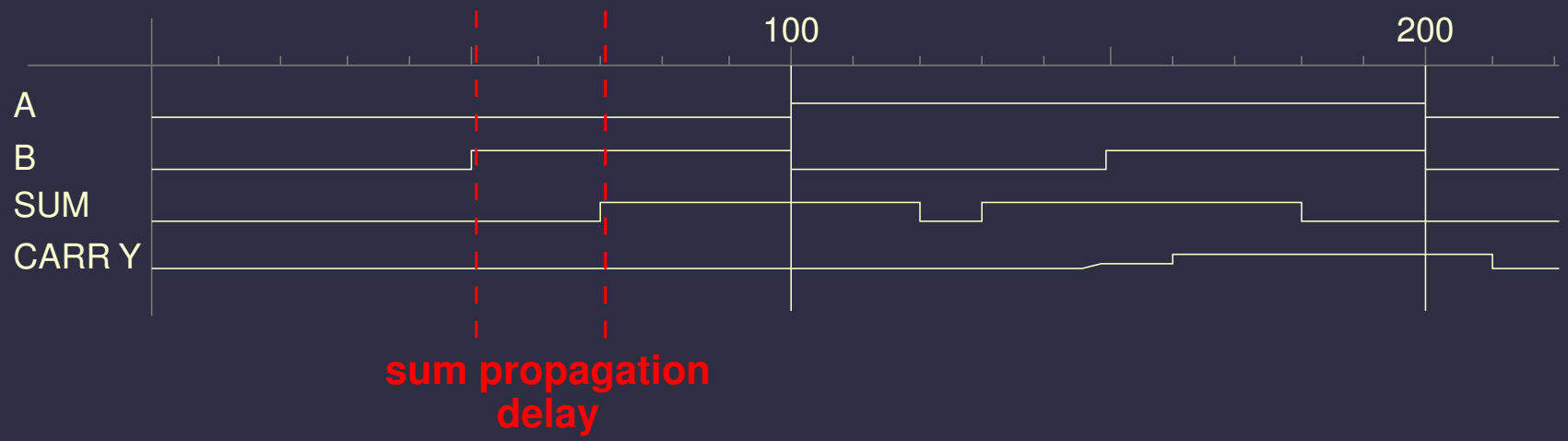
Lab one

- Walks you through the design and simulation of an exclusive-or (XOR) gate
- Due on 2 October
- Start early, especially if you are not familiar with Unix
- This lab requires a lot of tasks that are extremely easy the second time you do them, but slow and error-prone the first time through

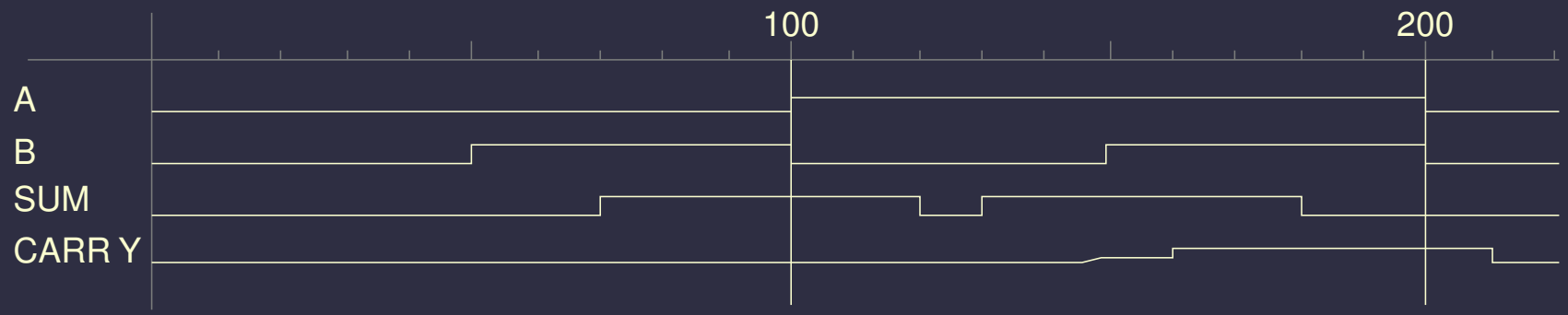
Section outline

2. Homework
    - Assignment
    - Details

Timing diagram for half adder

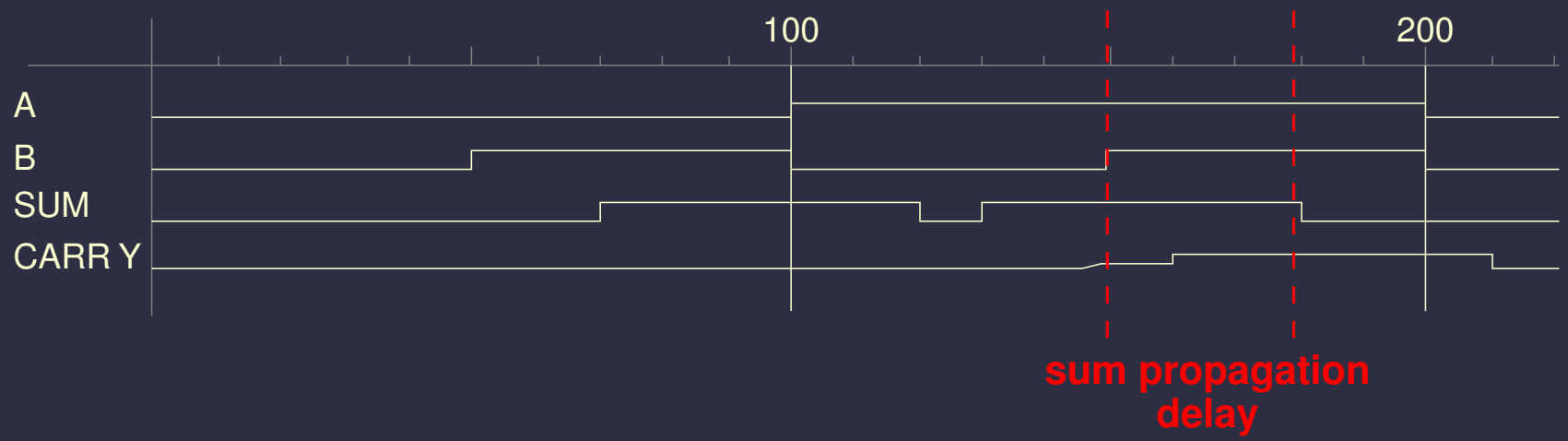


Timing diagram for half adder

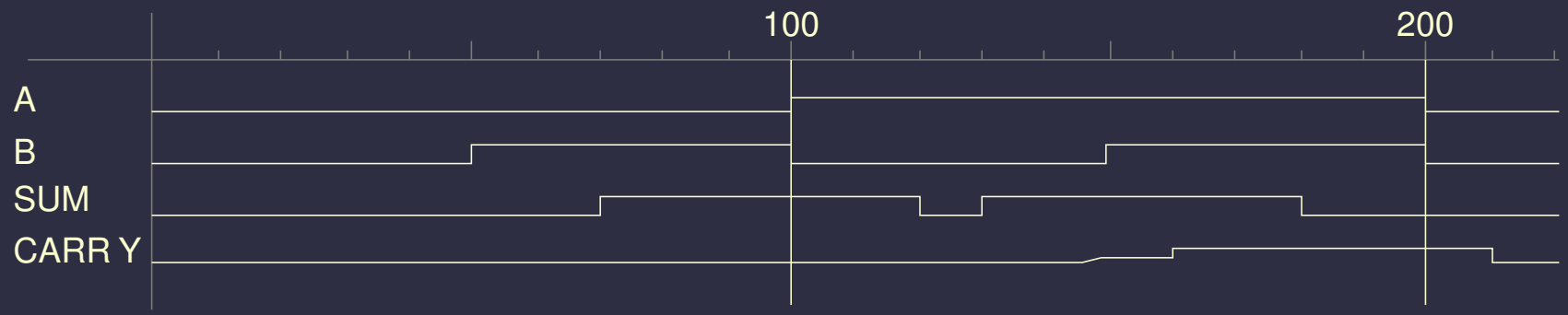


- Delay between input and output changes
- Delay is sensitive to circuit path

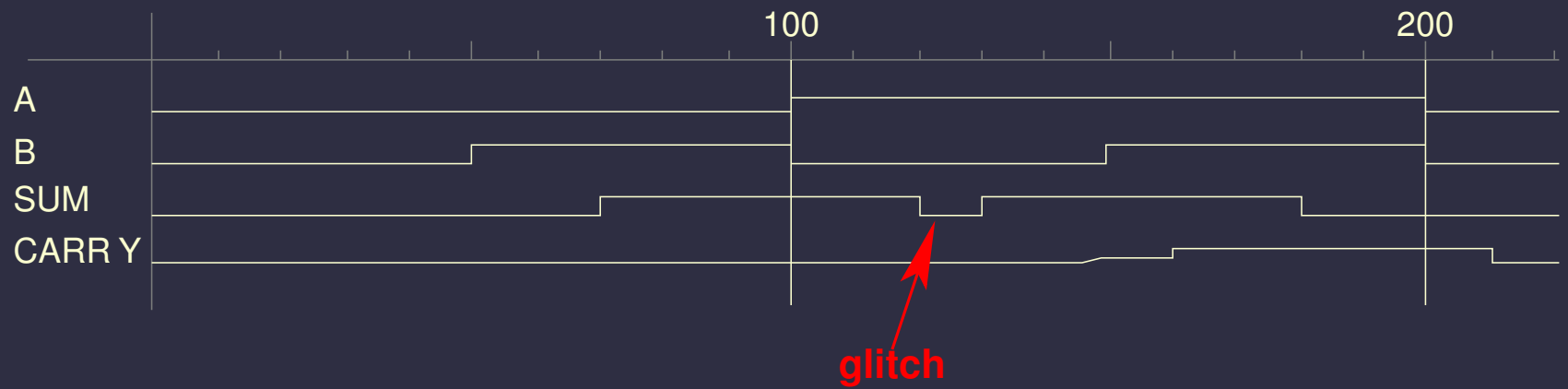
Timing diagram for half adder



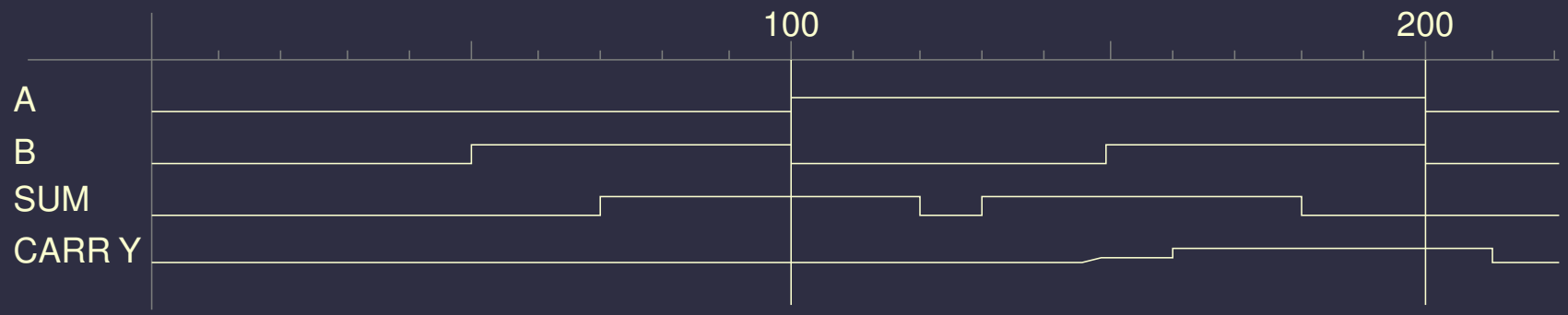
Timing diagram for half adder



Timing diagram for half adder

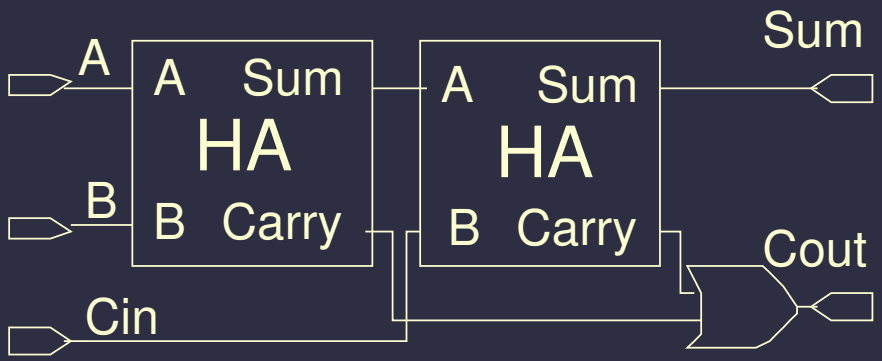


Timing diagram for half adder

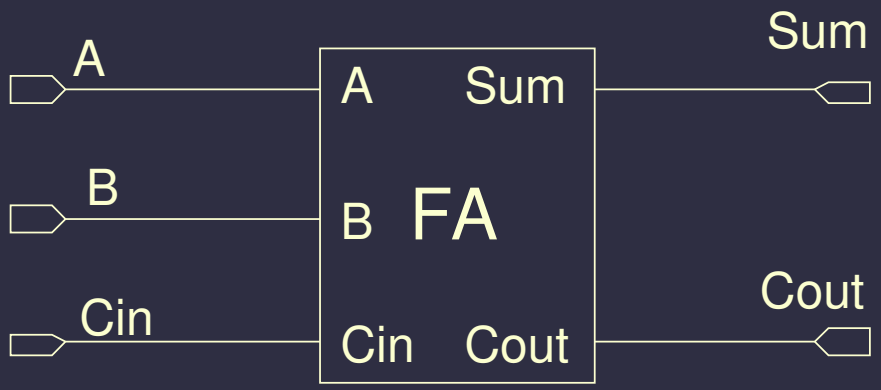


- Outputs may temporarily be incorrect before stabilizing
  - Glitches – caused by hazards

Block diagrams



Full adder composed of half adder blocks



Full adder block

- Structural organization of the design
- Hierarchical functional black boxes with input/output connections
- Concentrates on how the components are organized by wiring

Mentor Graphics tools introduction

- The Mentor Graphics CAD system has many components
- You will use a portion of the tools in this course
  - Falcon Design Framework
  - Design Architect for entering logic designs
  - Quicksim for simulating the designs
  - QuickHDL for entering and simulating the VHDL designs
- You will soon be working on lab 1, a Mentor Graphics tutorial

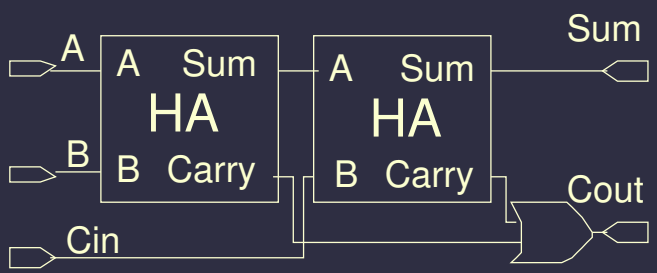
Mentor Graphics introduction

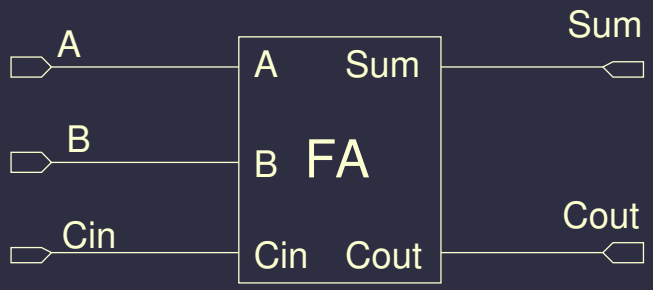
- Typing "source /vol/ece303/mgc.env" in the ECE filesystem will set up environment for ECE 303 labs
- Typing "dmgr" for Design Manager will open a window allowing several other Mentor Graphics to be run
- Mentor Graphics is not a single tool tool but a series of design tools that uses object oriented data representation to simplify the design process

Mentor Graphics introduction

- Data created in one tool (e.g., Design Architect) can be exported to another tool (e.g., Quicksim) for simulation
- A schematic is a diagram of a circuit
- Warning: Don't use OS commands to move directories or files
  - Design Manager needs to update other files when things are moved

Component definition





Data created by Design Architect is saved as components

- Models describing functional and graphical aspects
- Component data is composed of a schematic and a symbol
- A symbol is a graphical model with input and output pins
- A schematic is a functional model describing the relationship between output and input values

Viewpoint definition

- A viewpoint is a set of rules specifying a design's configuration
- Specifies the subset of data to use as input for a specific tool
- Allows a block to be described in different ways within different viewpoints
  - One can evaluate the impact of a low-level design decision on high-level design
  - For now, one viewpoint per design should be sufficient

Outline

Section outline

3. Two-level logic
   - Boolean algebra
   - Simplification
   - Karnaugh maps

Boolean algebra

- Set of elements, B
- Binary operators, { [AND, $\wedge$, *, $\cdot$], [OR, $\vee$, $+$] }
  - We'll prefer $\cdot$ and $+$
  - $\cdot$ frequently omitted
- Unary operator, [NOT, ', $\overline{o}$]

Axioms of Boolean algebra

$$\exists x, y \in B \text{ s.t. } x \neq y$$

closure	$\forall x, y \in B$	$xy \in B$
		$x + y \in B$
commutative laws	$\forall x, y \in B$	$xy = yx$
		$x + y = y + x$
identities	$0, 1 \in B, \forall x \in B$	$x1 = x$
		$x + 0 = x$

Axioms of Boolean algebra

$$\exists x, y \in B \text{ s.t. } x \neq y$$

distributive laws	$\forall x, y, z \in B$	$x + (yz) = (x + y)(x + z)$
		$x(y + z) = xy + xz$
compliment	$x \in B$	$x\overline{x} = 0$
		$x + \overline{x} = 1$

DeMorgan's laws

$$\overline{(a+b)} = \overline{a}\ \overline{b}$$
$$\overline{ab} = \overline{a} + \overline{b}$$
$$\overline{f(x_1, x_2, \ldots, x_n, \cdot, +)} = f(\overline{x_1}, \overline{x_2}, \ldots, \overline{x_n}, +, \cdot)$$

- Those xs could be functions
- Apply in stages
  - Top-down

DeMorgan's laws example

$$\overline{a + bc}$$

$$\overline{a} \cdot \overline{(bc)}$$

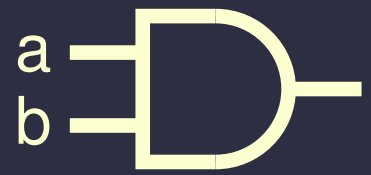
$$\overline{a} \cdot (\overline{b} + \overline{c})$$

Representations of Boolean functions

- Truth table
- Expression using only $\cdot$, $+$, and $'$
- Symbolic
- Karnaugh map
  - More useful as visualization and optimization tool

AND

a	b	a b
0	0	0
0	1	0
1	0	0
1	1	1



a *AND* b = a b

Will show Karnaugh map later

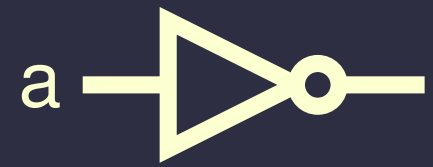
OR

a	b	a + b
0	0	0
0	1	1
1	0	1
1	1	1

$$a \; \mathit{OR} \; b = a + b$$

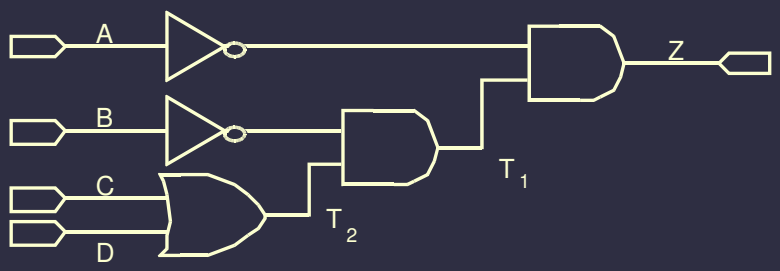
NOT

a	$\bar{a}$
0	1
1	0

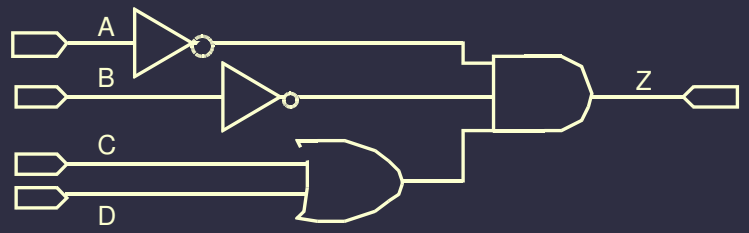


$$NOT\ \mathrm{a} = \bar{\mathrm{a}}$$

Different representations possible

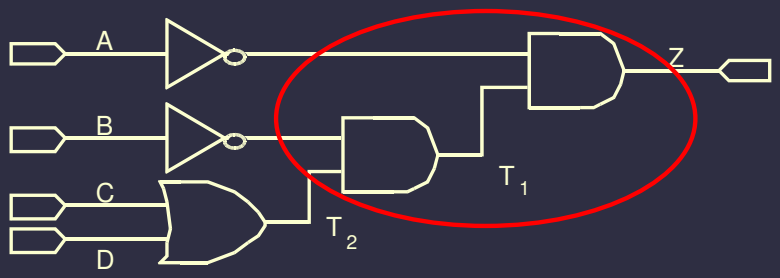


$$Z = ((C + D)\,\overline{B}\,)\,\overline{A}$$

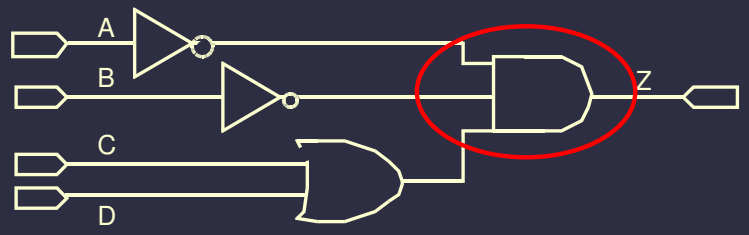


$$Z = (C + D)\,\overline{A}\;\overline{B}$$

Different representations possible



$$Z = ((C + D)\,\overline{B}\,)\,\overline{A}$$



$$Z = (C + D)\,\overline{A}\;\overline{B}$$

Section outline

3. Two-level logic
   - Boolean algebra
   - Simplification
   - Karnaugh maps

Simplifying logic functions

- Minimize literal count (related to gate count, delay)
- Minimize gate count
- Minimize levels (delay)
- Trade off delay for area
  - Sometimes no real cost

Simplifying logic functions

- Minimize literal count (related to gate count, delay)
- Minimize gate count
- Minimize levels (delay)
- Trade off delay for area
  - Sometimes no real cost

Proving theorems = simplification

Prove $XY + X\overline{Y} = X$

$$XY + X\overline{Y} = X(Y + \overline{Y}) \quad \text{distributive law}$$
$$X(Y + \overline{Y}) = X(1) \quad \text{complementary law}$$
$$X(1) = X \quad \text{identity law}$$

Proving theorems = simplification

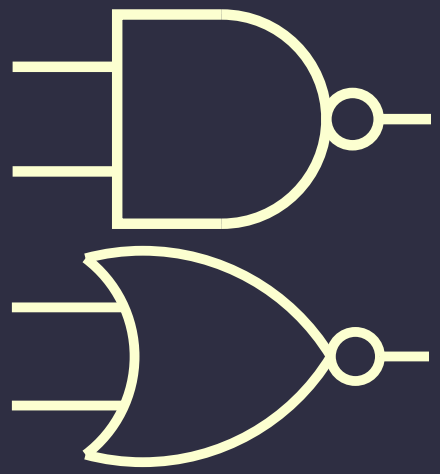
Prove $X + XY = X$

$$X + XY = X1 + XY \quad \text{identity law}$$
$$X1 + XY = X(1 + Y) \quad \text{distributive law}$$
$$X(1 + Y) = X1 \quad \text{identity law}$$
$$X1 = X \quad \text{identity law}$$

Literals

- Each appearance of a variable (complement) in expression
- Fewer literals usually implies simpler to implement
- E.g., $Z = A\overline{B}\,C + \overline{A}\,B + \overline{A}\,B\overline{C} + \overline{B}\,C$
  - Three variables, ten literals

NANDs and NORs



- Can be implemented in CMOS
  - More on this later
- X NAND $Y = \overline{XY}$
- X NOR $Y = \overline{X + Y}$
- Do we need inverters?

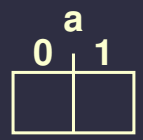
Section outline

3. Two-level logic
   - Boolean algebra
   - Simplification
   - Karnaugh maps

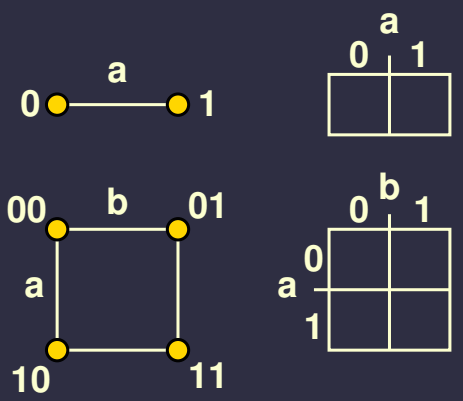
Karnaugh maps (K-maps)

- Fundamental attribute is adjacency
- Useful for logic synthesis
- Helps logic function visualization

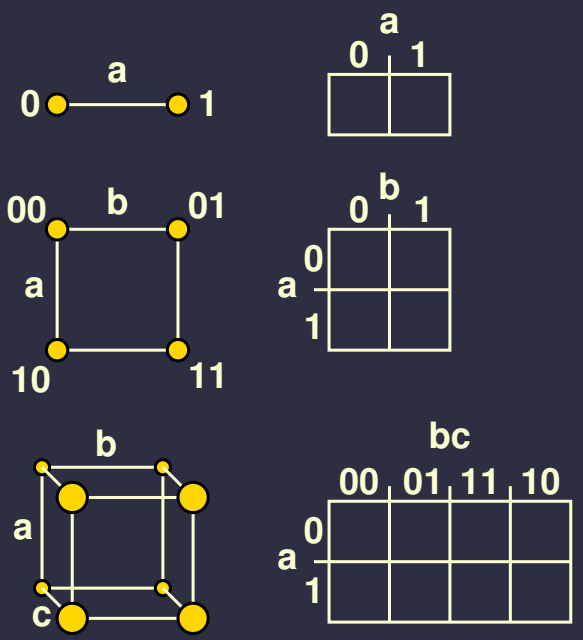
Karnaugh maps



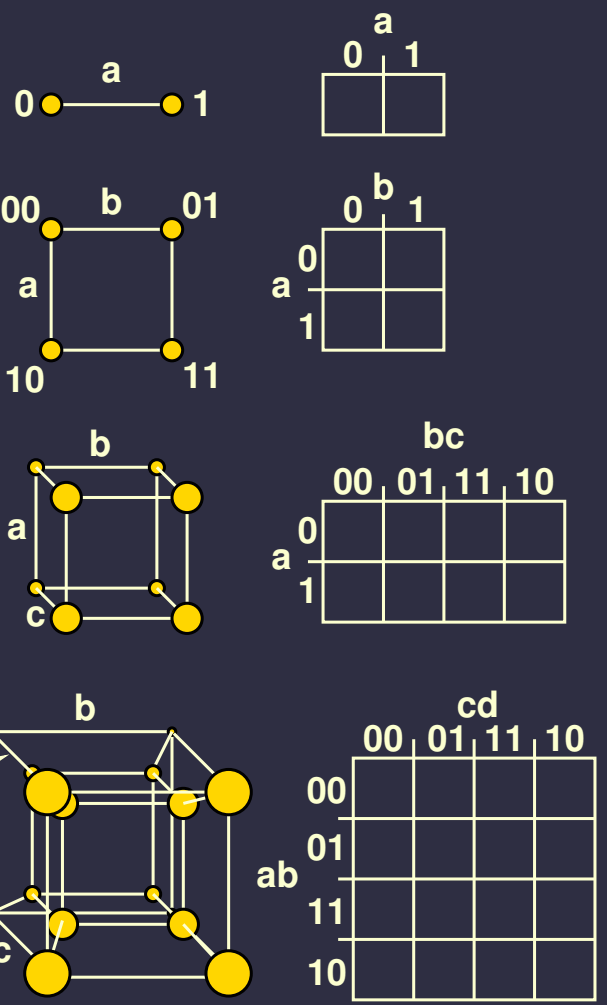
Karnaugh maps



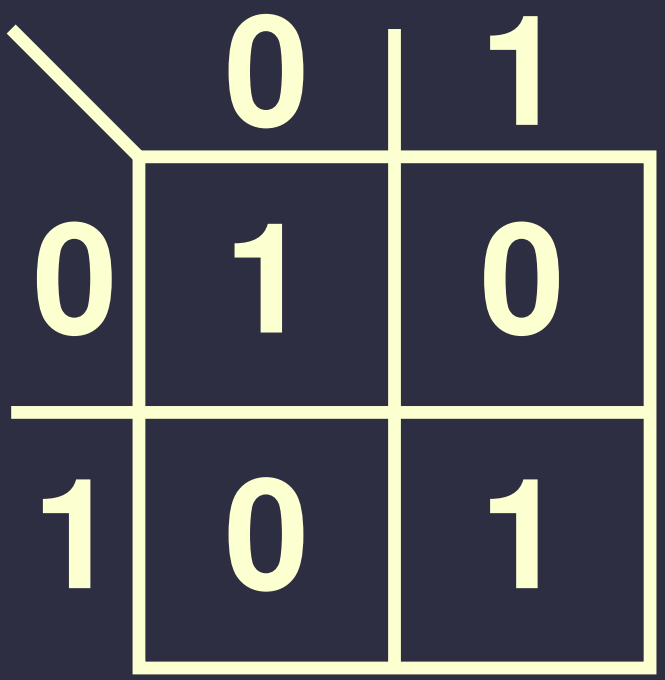
Karnaugh maps



Karnaugh maps



Sum of products (SOP)



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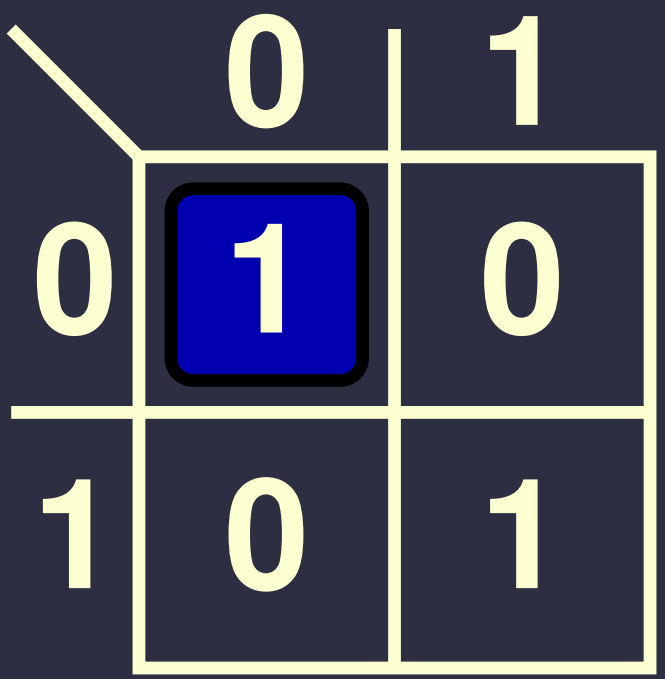
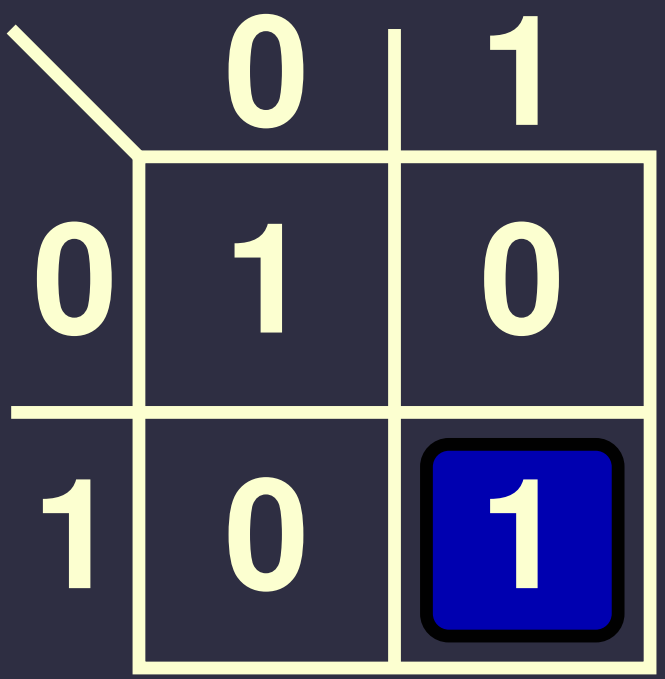


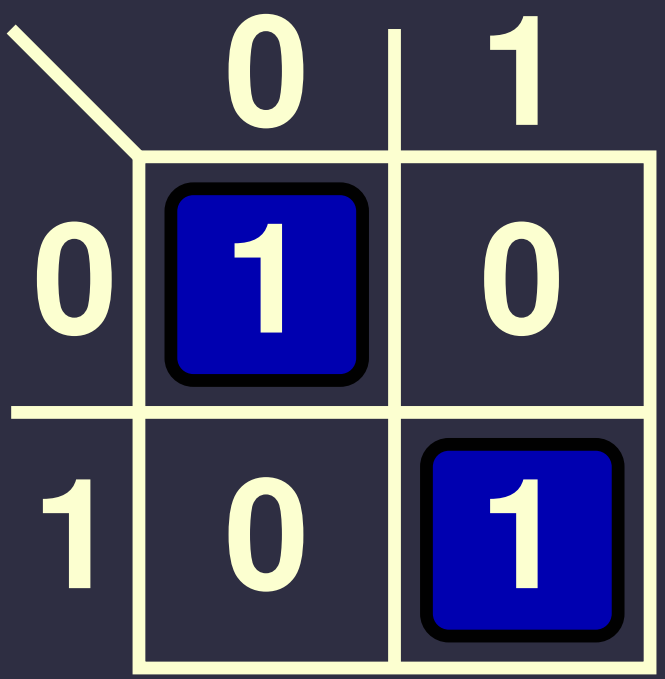
$(\overline{a}\ \overline{b})$

Sum of products (SOP)



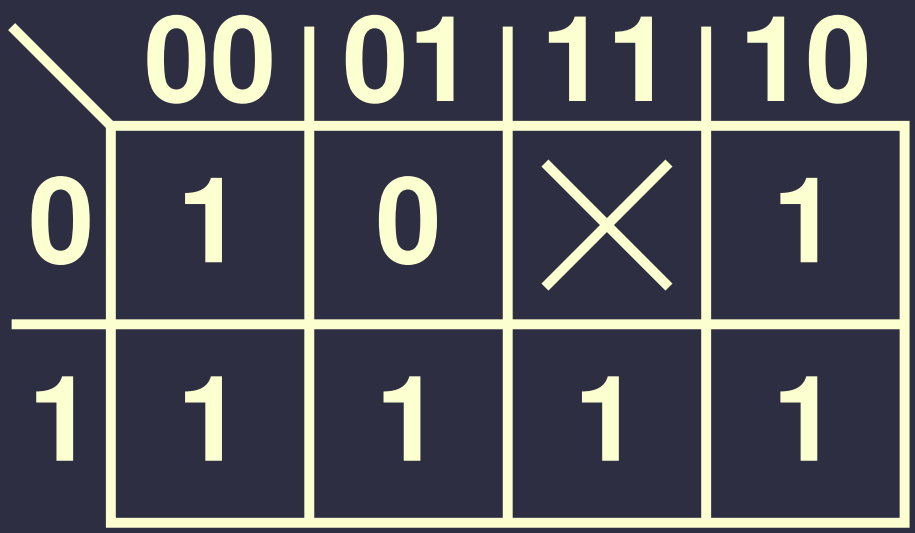
$(\overline{a}\ \overline{b})$ $(a\ b)$

Sum of products (SOP)

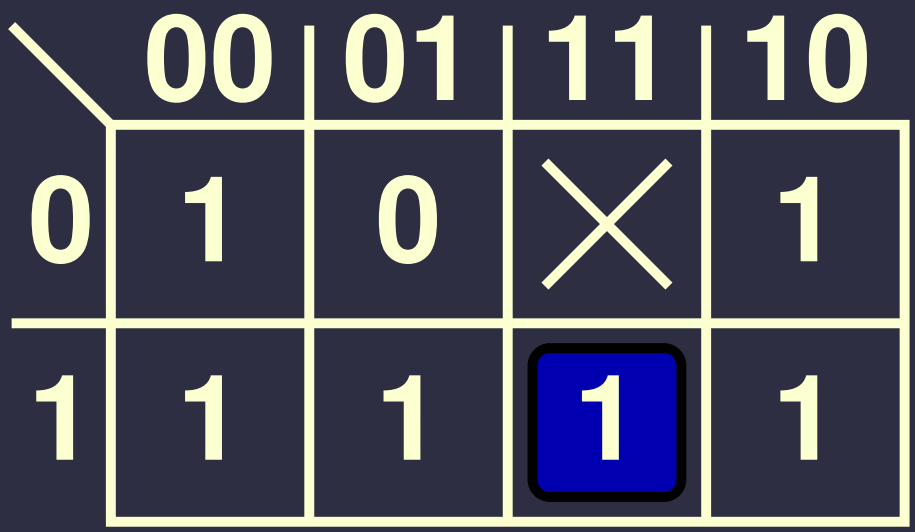


$$(\overline{a}\ \overline{b}) + (a\ b)$$

Implicants

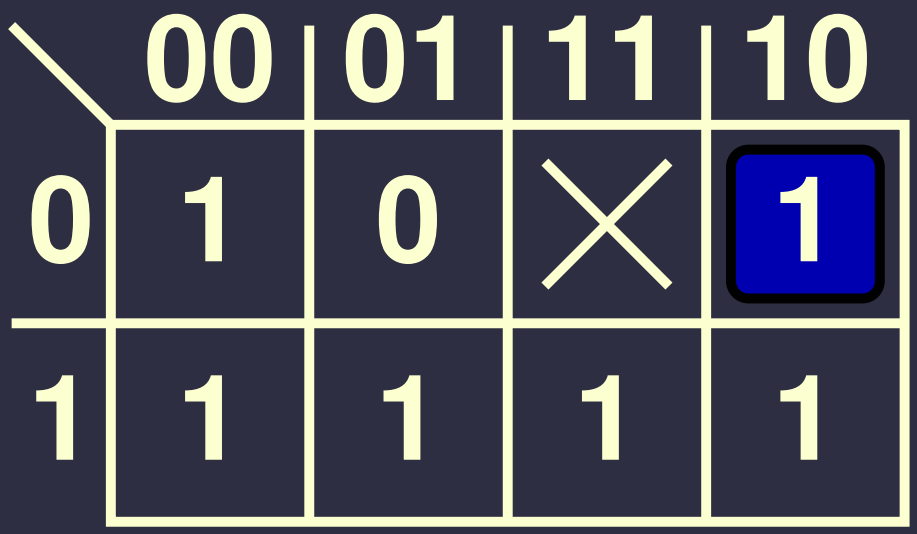


Implicants



implicant

Implicants



implicant

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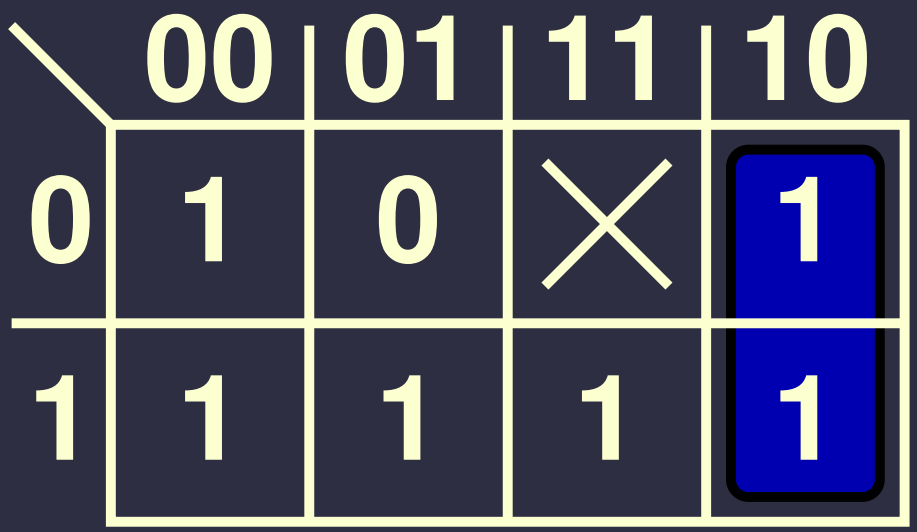
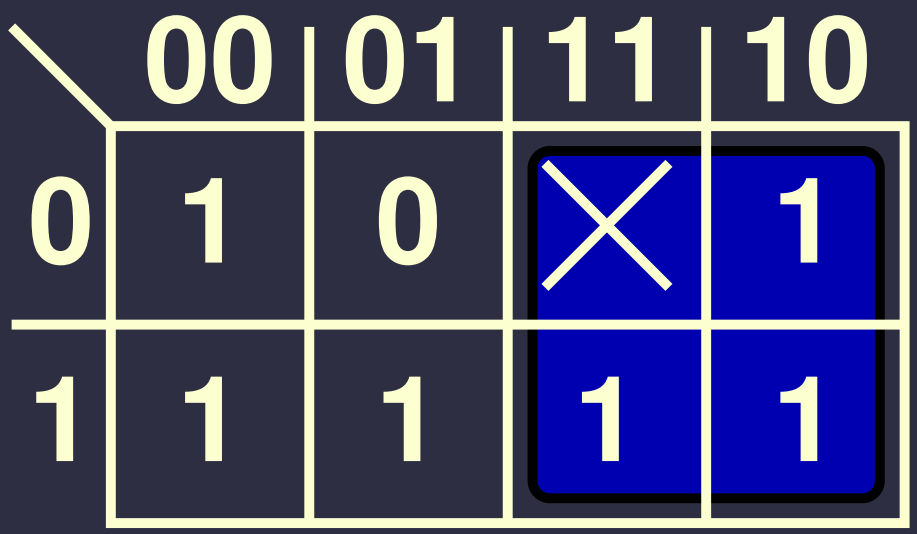


implicant

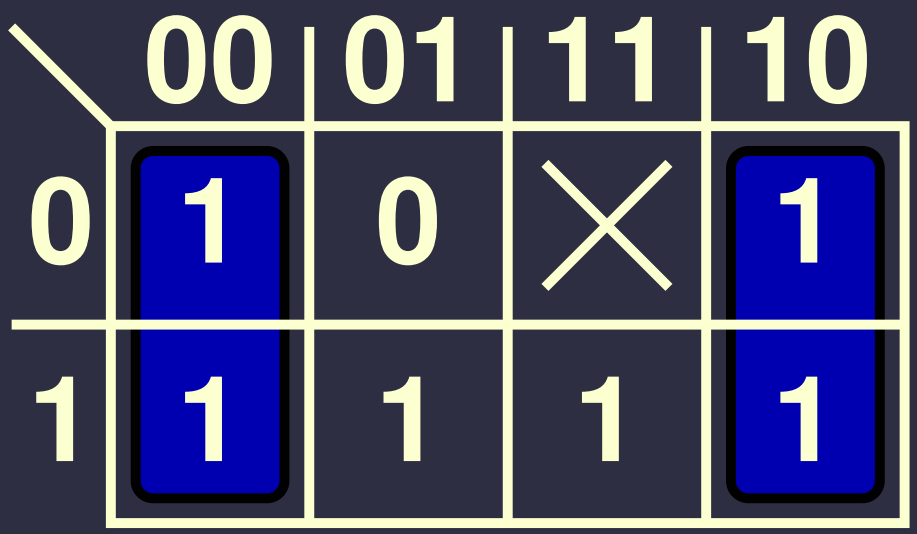
Implicants



prime implicant

Prime implicants are not covered by other implicants

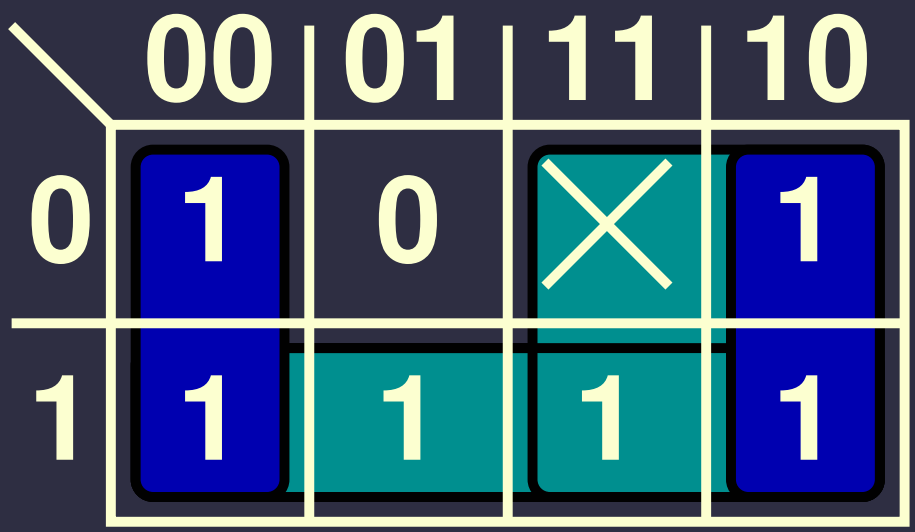
Implicants



essential prime implicant

Essential prime implicants uniquely cover minterms

Implicants



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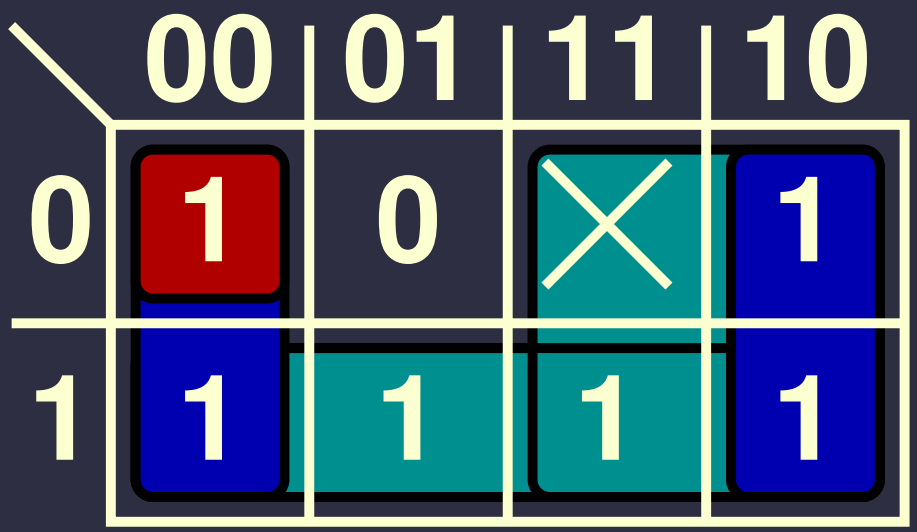


K-map example

- Minimize $f(a, b, c, d) = \sum(1, 3, 8, 9, 10, 11, 13)$

K-map example

- Minimize $f(a, b, c, d) = \sum(1, 3, 8, 9, 10, 11, 13)$
- f(a, b, c, d) = $a\ \overline{b} + \overline{b}\ \ d + a\ \overline{c}\ \ d$

K-map simplification technique

For all minterms

- Find maximal groupings of 1's and X's adjacent to that minterm.
- Remember to consider top/bottom row, left/right column, and corner adjacencies.
- These are the prime implicants.

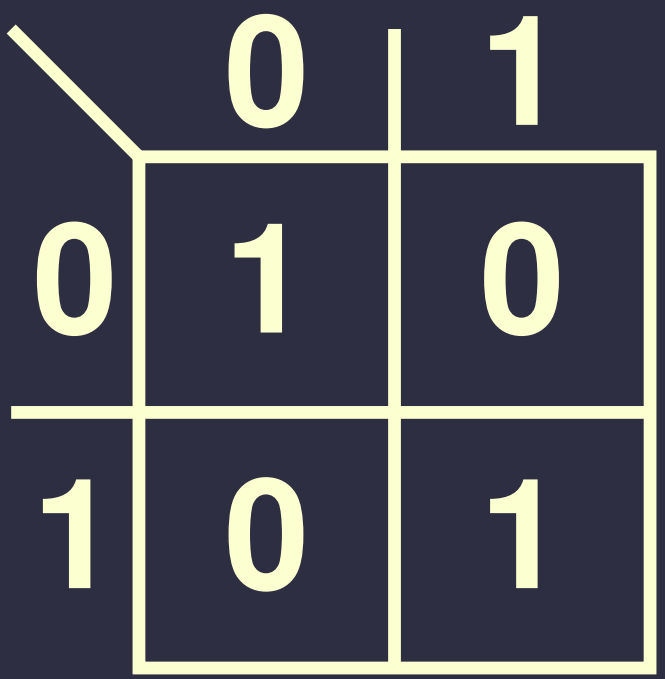
K-map simplification technique

- Revisit the 1's elements in the K-map.
- If covered by single prime implicant, the prime is essential, and participates in final cover.
- The 1's it covers do not need to be revisited.

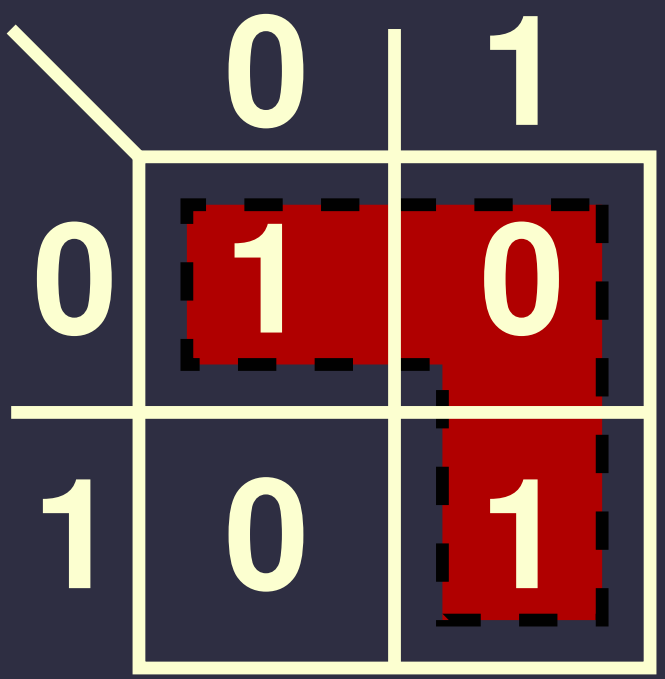
K-map simplification technique

- If there remain 1's not covered by essential prime implicants,
- Then select the smallest number of prime implicants that cover the remaining 1's.
- This can be difficult for complicated functions.
- Will present an algorithm for this in a future lecture.

Product of sums (POS)

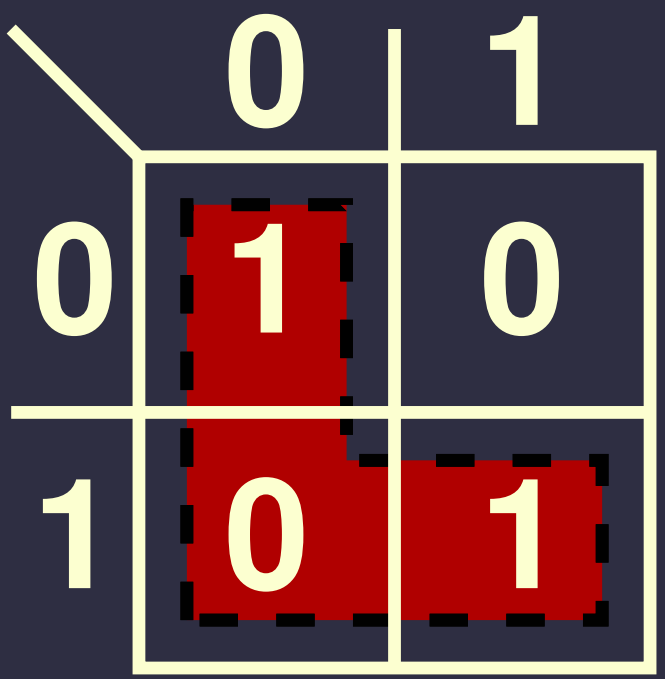


Product of sums (POS)



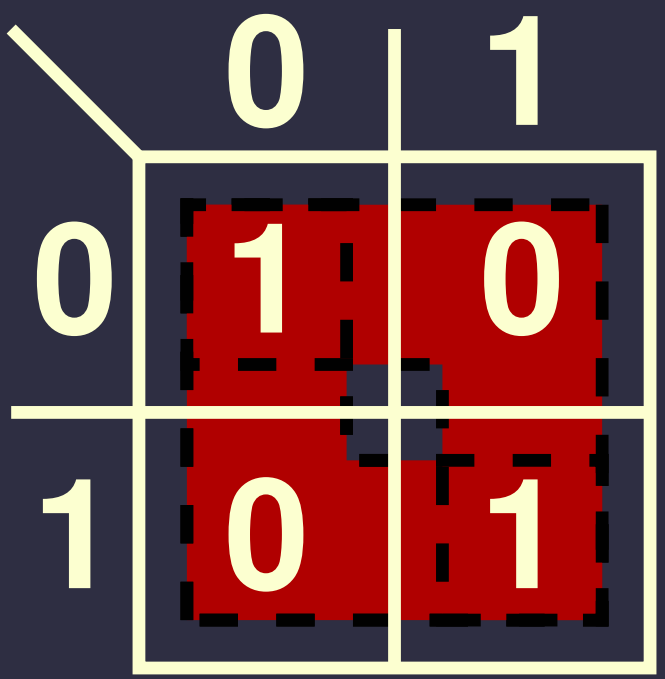
$(\overline{a} + b)$

Product of sums (POS)



$$(a + \overline{b})$$

Product of sums (POS)



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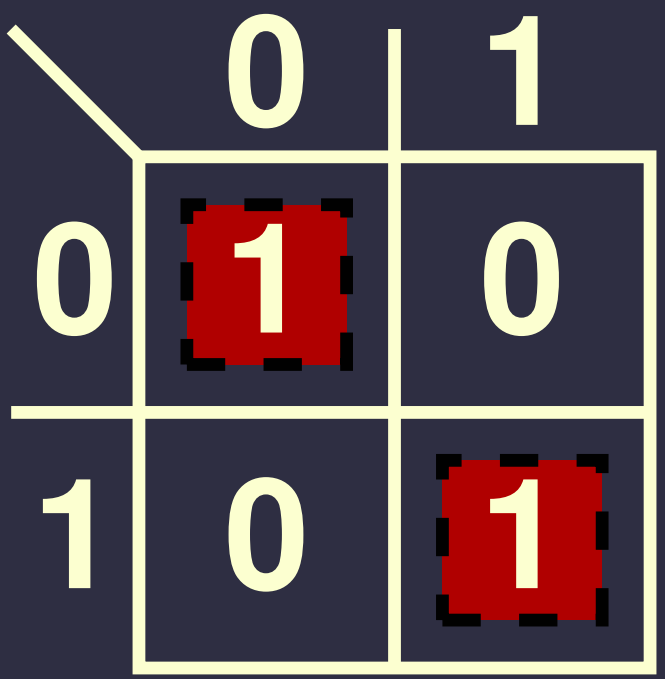


$$(\overline{a} + b) \cdot (a + \overline{b})$$

POS K-map techniques

- Direct reading by covering zeros and inverting variables

Or

- Invert function
- Do SOP
- Invert again
- Apply DeMorgan's laws

POS K-map example

- Minimize $f(a, b, c) = \prod(2, 4, 5, 6)$

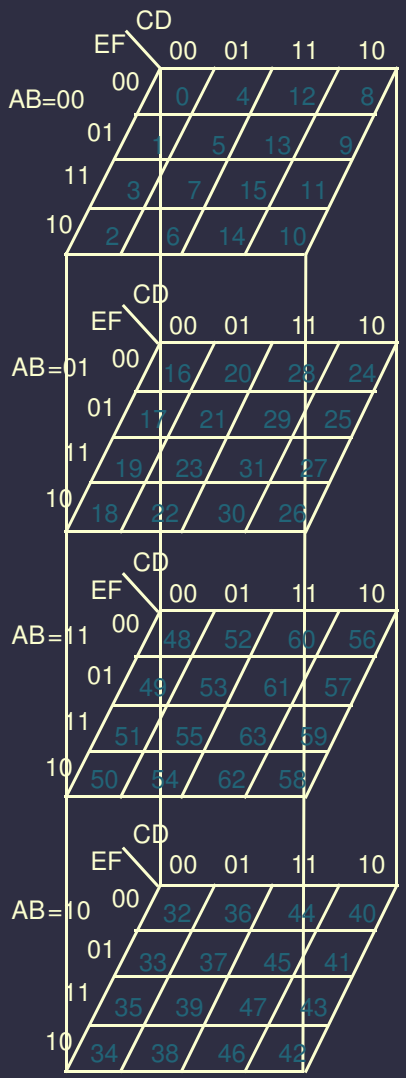
POS K-map example

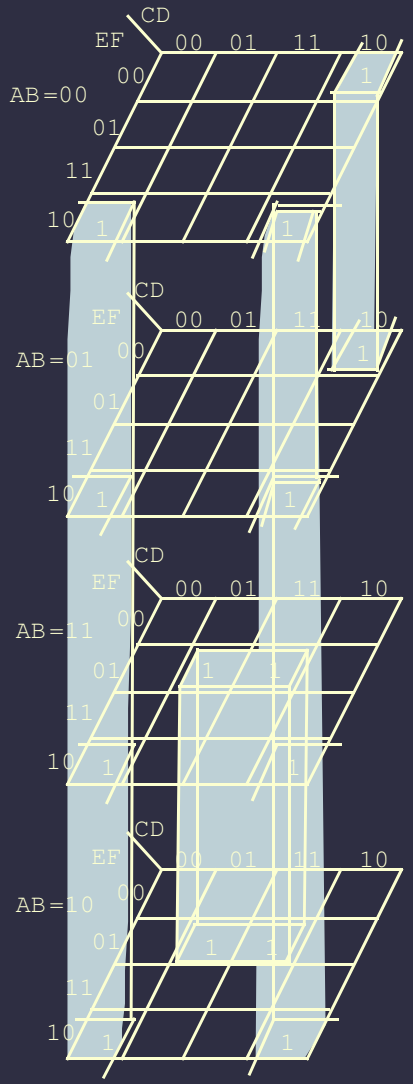
- Minimize $f(a, b, c) = \prod(2, 4, 5, 6)$
- $f(a, b, c) = (\overline{b} + c)(\overline{a} + b)$

Six-variable K-map example

$$z(a, b, c, d, e, f) = \sum(2, 8, 10, 18, 24, 26, 34, 37, 42, 45, 50, 53, 58, 61)$$

Six-variable K-map example





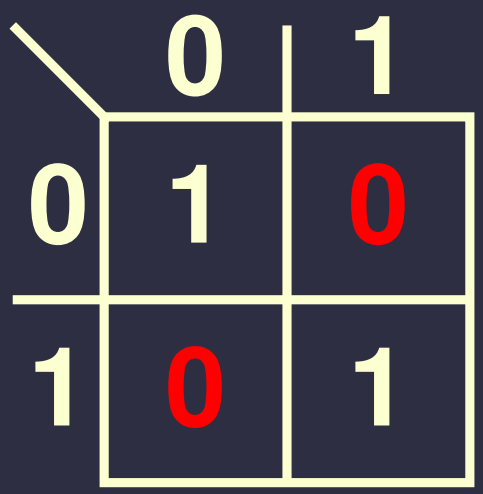
Six-variable K-map example

$$z(a,b,c,d,e,f) = \overline{d}\ e\,\overline{f} + ad\,\overline{e}\ f + \overline{a}\ C\,\overline{d}\ \overline{f}$$

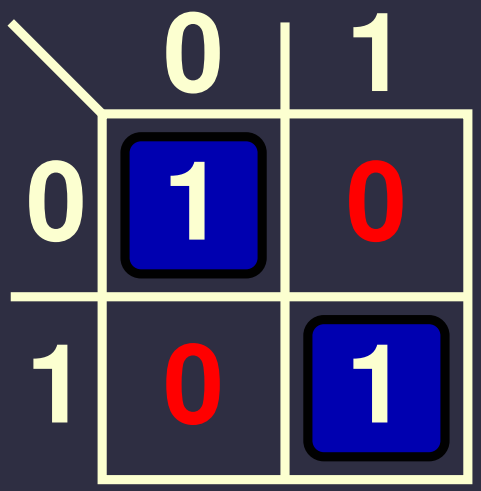
Don't Care logic

- All specified Boolean values are 0 or 1
- However, during design some values may be unspecified
  - Don't care values ($\times$)
- At $\times$s allow circuit optimization
  - Incompletely specified functions allow optimization

Don't Care values



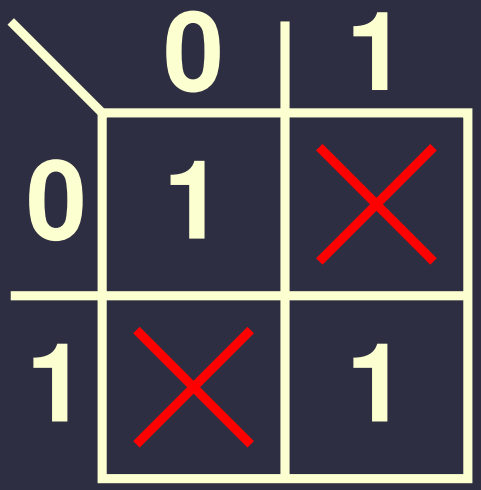
Don't Care values



Can let the undefined values be zero

- Correct...
- ...however, complicated
- $(\overline{a}\ \overline{b}) + (a\ b)$

Don't Care values



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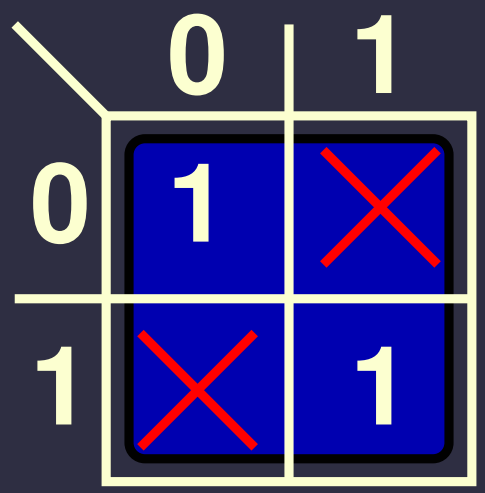
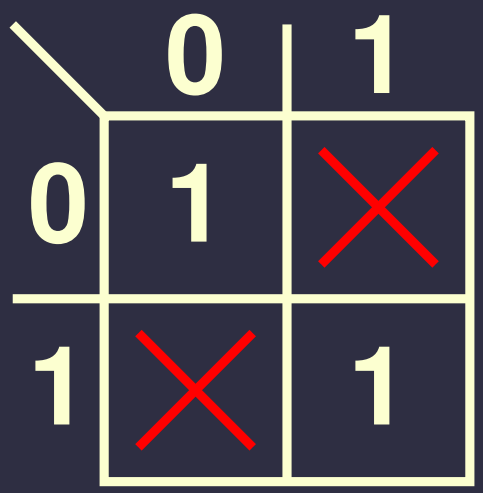
Don't Care values



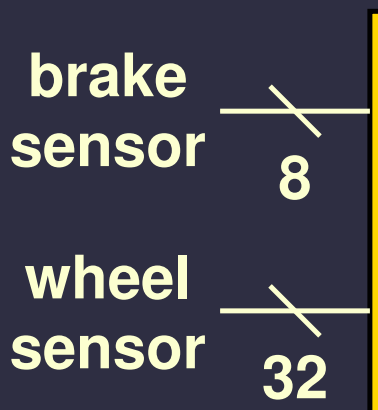
Instead, leave these values undefined ($\times$)

- Also called Don't Care values
- Allows any function implementing the specified values to be used
- E.g., could use $(\overline{a} \cdot \overline{b}) + (a \cdot b)$

Don't Care values



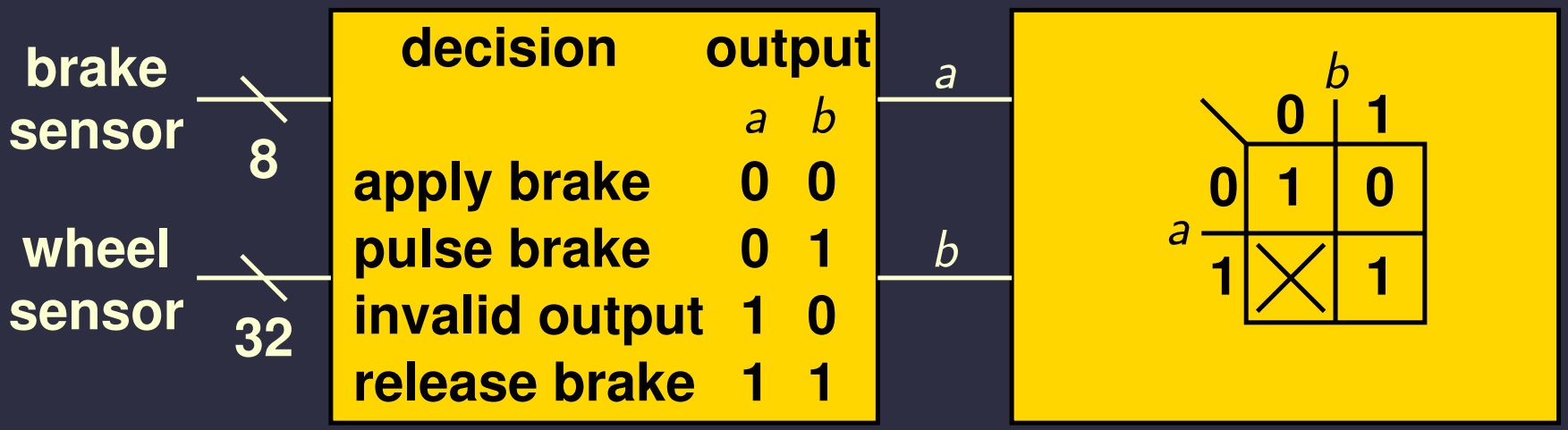
Satisfiability Don't Cares



decision	output	
	a	b
apply brake	0	0
pulse brake	0	1
invalid output	1	0
release brake	1	1

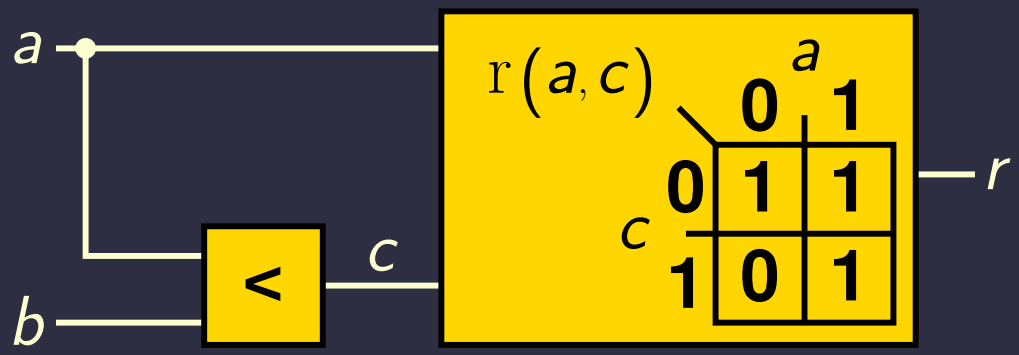
- Input can never occur
- This can happen within a circuit
- Some modules will not be capable of producing certain outputs

Satisfiability Don't Cares



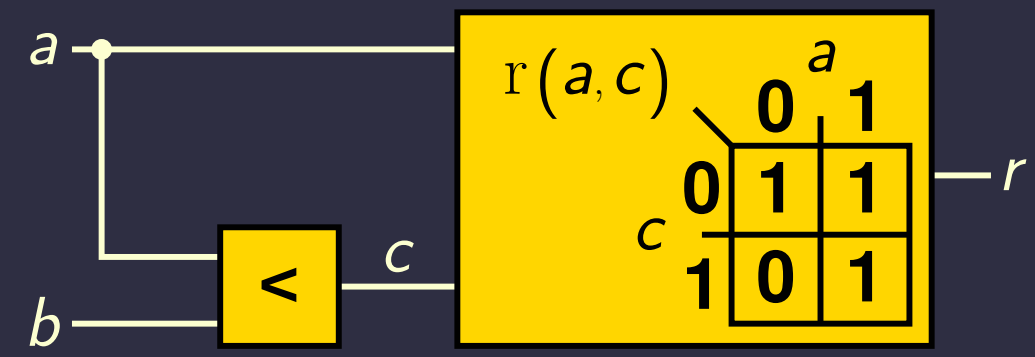
- Input can never occur
- This can happen within a circuit
- Some modules will not be capable of producing certain outputs

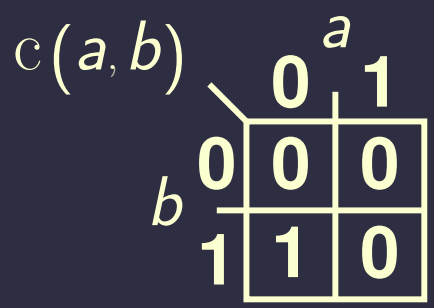
Observability Don't Cares



Output will be ignored for certain inputs

Observability Don't Cares





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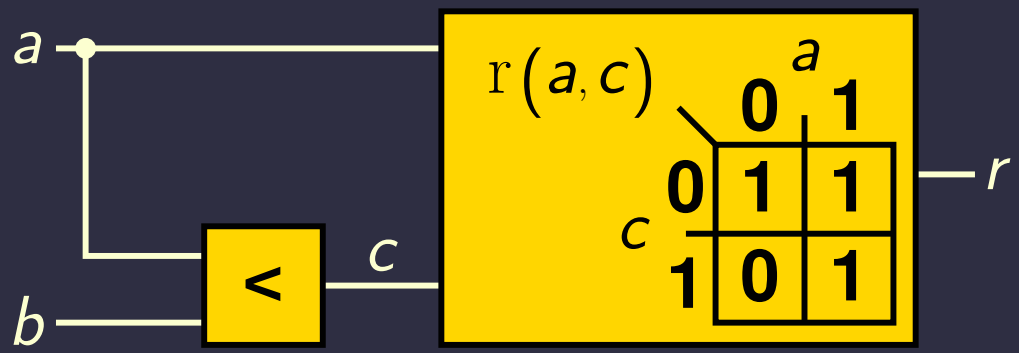
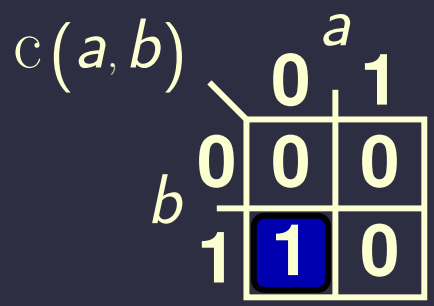
Observability Don't Cares

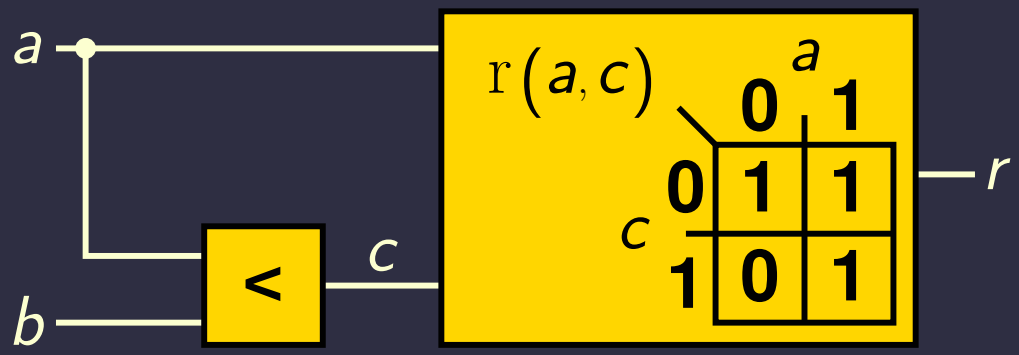


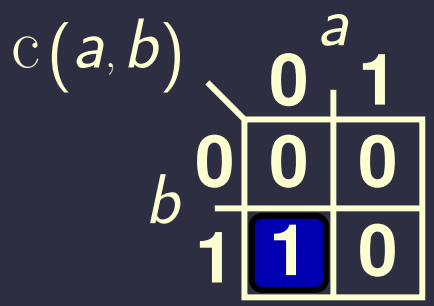


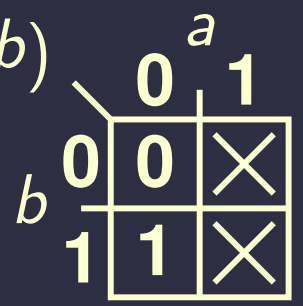
$$c(a,b) = \overline{a} \wedge b$$

Output will be ignored for certain inputs

Observability Don't Cares







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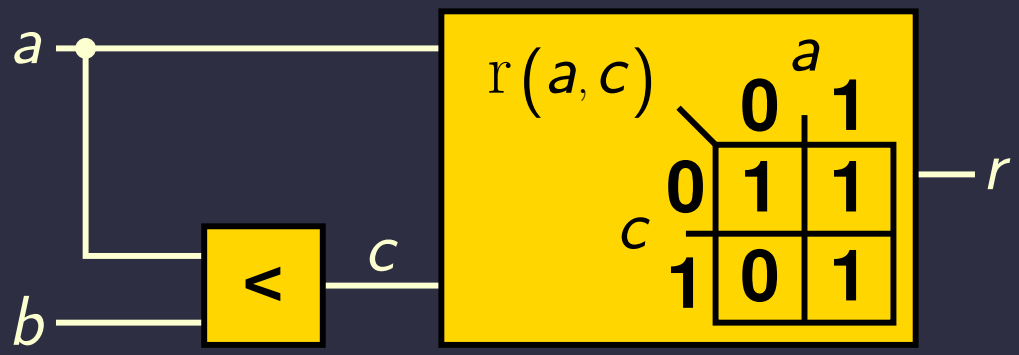
Observability Don't Cares



$c(a,b)$

b \ a	0	1
0	0	0
1	1	0

$$c(a,b) = \overline{a} \wedge b$$

$c'(a,b)$

b \ a	0	1
0	0	X
1	1	X

$$c'(a,b) = b$$

Output will be ignored for certain inputs

Don't-care K-map example

- Minimize $f(a, b, c, d) = \sum(1, 3, 8, 9, 10, 11, 13) + d(5, 7, 15)$

Don't-care K-map example

- Minimize $f(a, b, c, d) = \sum(1, 3, 8, 9, 10, 11, 13) + d(5, 7, 15)$
- $f(a, b, c, d) = a\ \overline{b} + d$

CMOS

- Refer to M. Morris Mano and Charles R. Kime. *Logic and Computer Design Fundamentals*. Prentice-Hall, NJ, fourth edition, 2008
- http://www.writphotec.com/mano/
- CMOS supplement
- Optimization supplement
- qm.py at http://ziyang.eecs.northwestern.edu/~dickrp/tools.html

Outline

Intractable problems reading assignment

- Michael R. Garey and David S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman & Company, NY, 1979
- Chapter 1, Sections 1–5
- Introduces the concept of intractable problems
- Many problems in digital design are intractable
  - Too hard to solve optimally in a reasonable amount of time
- Use heuristics

Computer Geek Culture

- Python and perl