# Embedded Systems: An Application-Centered Approach

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#### Outline

- 1. Optimization for synthesis
- 2. Homework

### Synthesis motivation

- Embedded systems are found everywhere: cars, houses, games, phones, hospitals, etc.
- Designers need tools to deal with increasing complexity, increase product quality, and guarantee correct operation.
- Software or hardware errors are not acceptable. Anti-lock brake systems aren't allowed to crash.
- Embedded systems should not require bug fixes or upgrades.
- Price competition can be intense.
- Power consumption should be low.

#### Optimization for synthesis Homework

## Allocation, assignment, and scheduling Brief introduction to NP-completeness Complete optimization/search Stochastic optimization techniques







#### Optimization for synthesis Homework

## Allocation, assignment, and scheduling Brief introduction to NP-completeness Complete optimization yearch Stochastic ontimization techniques









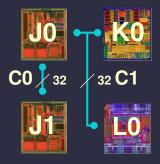


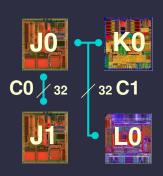


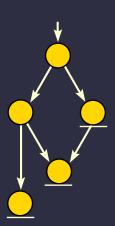


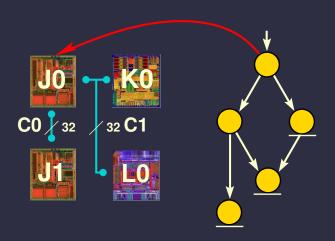


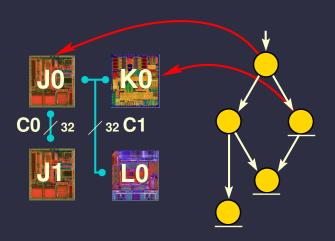


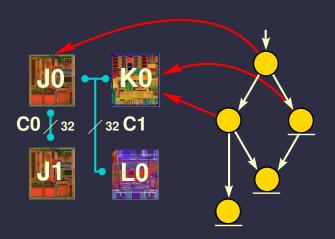


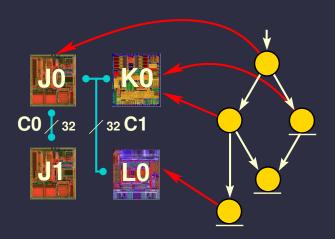


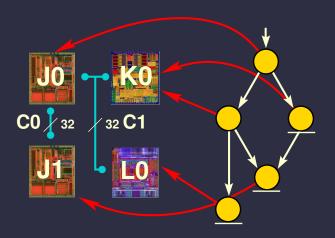


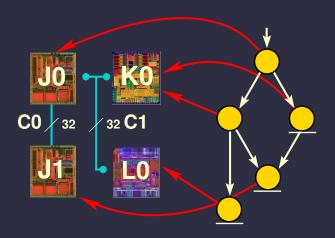


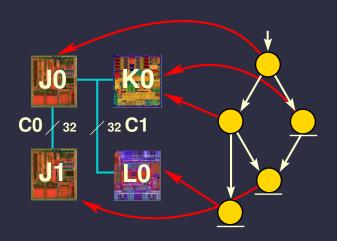


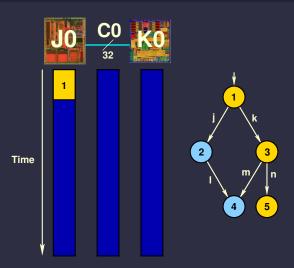


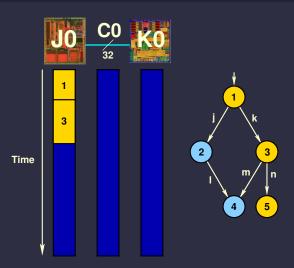


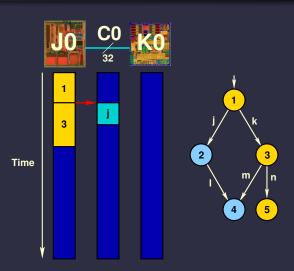


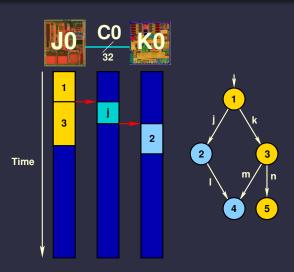


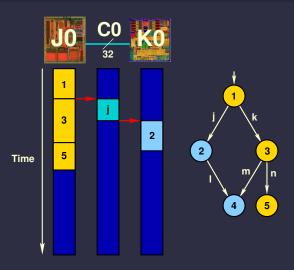


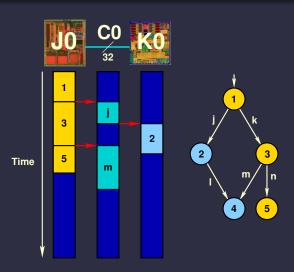


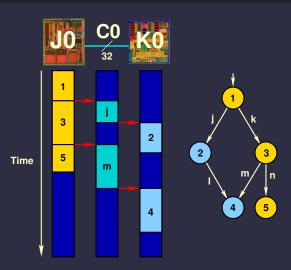


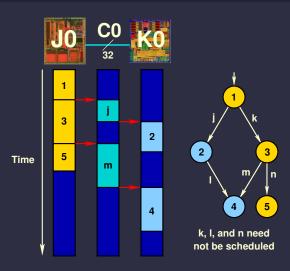






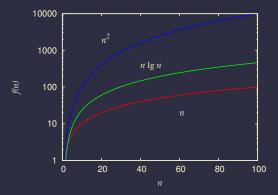






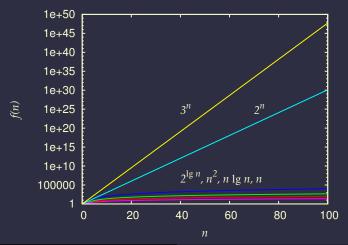
### Polynomial time complexities

- Recall that sorting may be done in  $\mathcal{O}(n \lg n)$  time
- DFS  $\in \mathcal{O}(|V| + |E|)$ , BFS  $\in \mathcal{O}(|V|)$
- Topological sort  $\in \mathcal{O}(|V| + |E|)$



#### Exponential time complexities

There also exist exponential-time algorithms:  $\mathcal{O}\left(2^{\lg n}\right)$ ,  $\mathcal{O}\left(2^{n}\right)$ ,  $\mathcal{O}\left(3^{n}\right)$ 



### Implications of exponential time complexity

For 
$$t(n) = 2^n$$
 seconds 
$$t(1) = 2 \text{ seconds}$$
 
$$t(10) = 17 \text{ minutes}$$
 
$$t(20) = 12 \text{ days}$$
 
$$t(50) = 35,702,052 \text{ years}$$
 
$$t(100) = 40,196,936,841,331,500,000,000 \text{ years}$$

### $\mathcal{NP}$ -complete problems

- Digital design and synthesis is full of NP-complete problems
- Graph coloring
- Allocation/assignment
- Scheduling
- Graph partitioning
- Satisfiability (and 3SAT)
- Covering
- ...and many more

### Conjecture on hardness of problems

- There is a class of problems,  $\mathcal{NP}$ -complete, for which nobody has found polynomial time solutions
- It is possible to convert between these problems in polynomial time
- Thus, if it is possible to solve any problem in  $\mathcal{NP}$ -complete in polynomial time, all can be solved in polynomial time
- $\mathcal{P} \subseteq \mathcal{NP}$
- Unproven conjecture:  $\mathcal{P} \neq \mathcal{NP}$



- What is  $\mathcal{NP}$ ? Nondeterministic polynomial time.
- A computer that can simultaneously follow multiple paths in a solution space exploration tree is nondeterministic. Such a computer can solve  $\mathcal{NP}$  problems in polynomial time.
- Nobody has been able to prove either

$$\mathcal{P} \neq \mathcal{NP}$$

or

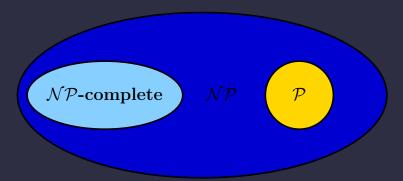
$$\mathcal{P} = \mathcal{N}\mathcal{P}$$

### $\mathcal{NP}$ -completeness

If we define  $\mathcal{NP}$ -complete to be a set of problems in  $\mathcal{NP}$  for which any problem's instance may be converted to an instance of another problem in  $\mathcal{NP}$ -complete in polynomial time, then

$$\mathcal{P} \subsetneq \mathcal{NP} \Rightarrow \mathcal{NP} ext{-complete} \cap \mathcal{P} = \varnothing$$

### Basic complexity classes



- $\mathcal{P}$  solvable in polynomial time by a computer (Turing Machine).
- ullet  $\mathcal{NP}$  solvable in polynomial time by a nondeterministic computer.
- $\mathcal{NP}$ -complete converted to other  $\mathcal{NP}$ -complete problems in polynomial time.

- What should you do when you encounter an apparently hard problem?
- Is it in  $\mathcal{N}\overline{\mathcal{P}}$ -complete?
- If not, solve it
- If so, then what?

Despair.

- What should you do when you encounter an apparently hard problem?
- Is it in  $\mathcal{NP}$ -complete?
- If not, solve it
- If so, then what?

Solve it!

- What should you do when you encounter an apparently hard problem?
- Is it in  $\mathcal{NP}$ -complete?
- If not, solve it
- If so. then what?

Resort to a suboptimal heuristic. Bad, but sometimes the only choice.

- What should you do when you encounter an apparently hard problem?
- Is it in  $\mathcal{N}\overline{\mathcal{P}}$ -complete?
- If not, solve it
- If so, then what?

Develop an approximation algorithm.

Better.

- What should you do when you encounter an apparently hard problem?
- Is it in  $\mathcal{NP}$ -complete?
- If not, solve it
- If so, then what?

Determine whether all encountered problem instances are constrained. Wonderful when it works.

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# One example

O. Coudert. Exact coloring of real-life graphs is easy. *Design Automation*, pages 121–126, June 1997.

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# Properties of complete optimization techniques

- If a solution exists, will be found
- Very slow for some problems
- Good formal understanding of complexity

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# Example complete algorithms

- Enumeration
- Branch and bound
- Dynamic programming
- Integer-linear programming
- Backtracking iterative improvement

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Stochastic optimization techniques

- Considers all possible solutions
- Extremely slow for large n
- Potentially has low constant factor, may be O.K. for small n

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# Example problem

Traveling salesman problem

Find shortest path visiting all cities.







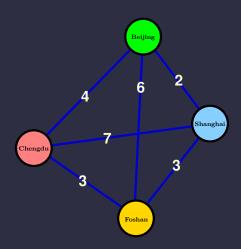
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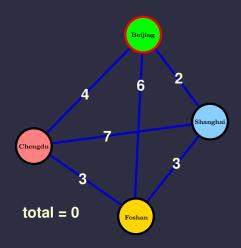




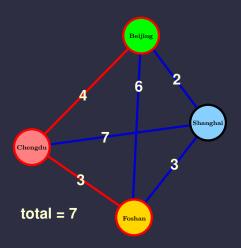


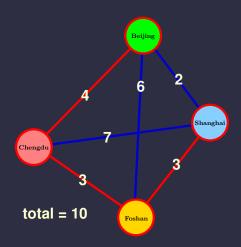


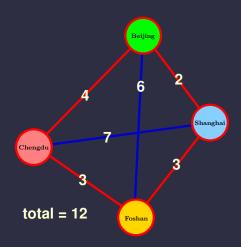






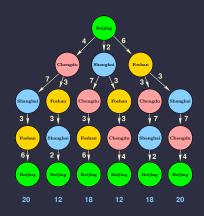








$$t(n) \ge c \cdot 2^n$$
 for  $n > n_0$ 



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- Keep track of minimal encountered cost
- When a path has a higher cost, terminate

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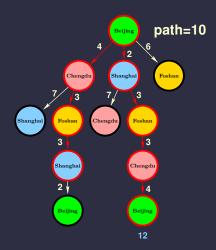


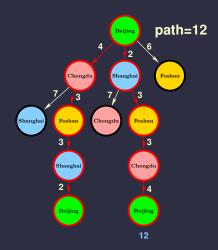




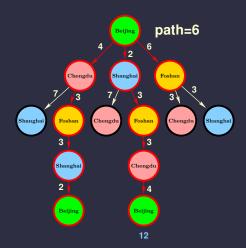




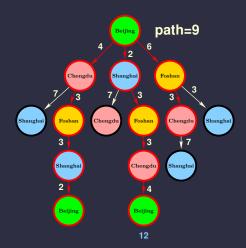


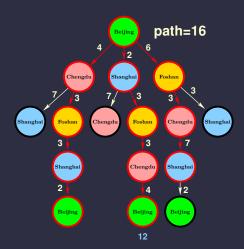


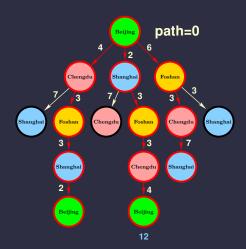


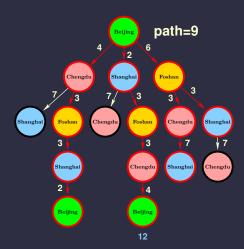


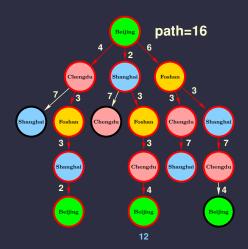
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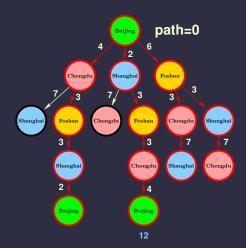


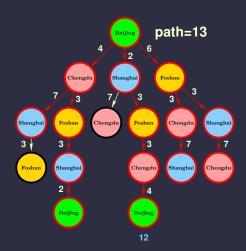


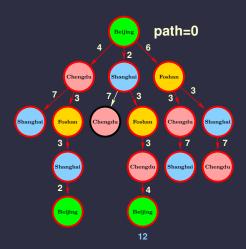


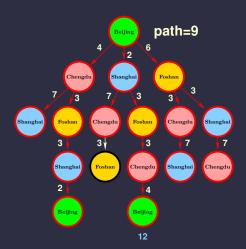






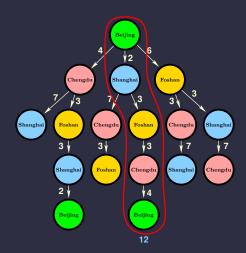












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- Better average-case complexity
- Still worst-case exponential

# Linear programming

- In  $\mathcal{P}-$  Ellipsoid Algorithm / internal point methods
- However, in practice WC exponential Simplex Algorithm better
- Goal: Maximize a linear weighted sum under constraints on variables

# Linear programming

Maximize

$$c_1 \cdot x_1 + c_2 \cdot x_2 + \cdots + c_n \cdot x_n$$

where

$$\forall c_i \in c, c_i \in R$$

subject to the following constraints:

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \dots + a_{1n} \cdot x_n \le =, \ge b_1$$
  
 $a_{21} \cdot x_1 + a_{22} \cdot x_2 + \dots + a_{2n} \cdot x_n \le =, \ge b_2$   
....

$$a_{n1} \cdot x_1 + a_{1n} \cdot x_2 + \dots + a_{nn} \cdot x_n \le = \ge b_n$$
  
 $\forall x_i \in x, x_i \ge 0$   $\forall a_{jk} \in A, a_{jk} \in R$ 

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# Linear programming

- · Can be formulated as a linear algebra problem
  - Vector x of variables
  - Vector c of cost
  - Matrix A of constraints
  - Vector b of constraints
- Maximize or minimize  $c^T x$
- Satisfy  $Ax \leq b$
- Satisfy  $x \ge 0$

# Integer-linear programming (ILP)

- ILP is  $\mathcal{NP}$ -complete
- LP with some variables restricted to integer values
- Formulate problem as ILP problem
  - Excellent understanding of problem
  - Good solvers exist
- Variants both NP-complete
  - Mixed ILP has some continuous variables
  - Zero-one ILP

# Example – ILP formulation for the travaling salesman problem

Let T be a tentative solution, or tour  $\forall e \in E$  let there be a variable

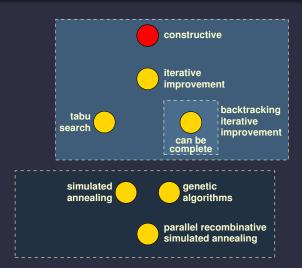
$$t_e = egin{cases} 1 & ext{if } e \in T \ 0 & ext{if } e 
otin T \end{cases}$$

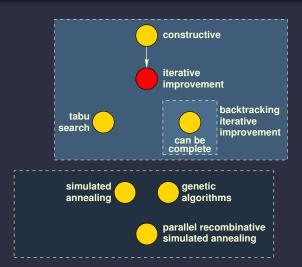
Constraint: Given that S is a set of vertices,  $\mathbf{con}(S)$  is the set of edges connecting  $v \in S$  to  $v \notin S$ , and  $\{v_i\}$  is the vertex set containing only  $v_i$ , every vertex,  $v_i$  must be connected to two edges of the tour

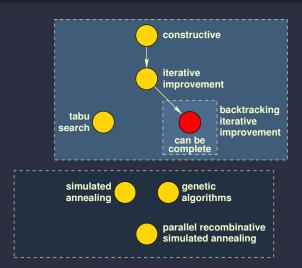
$$\forall v_i \in V, \sum_{e \in \mathsf{con}(\{v_i\})} = 2$$

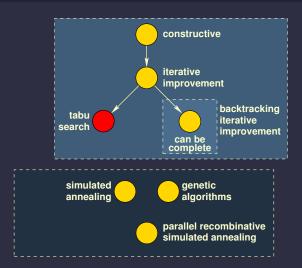
# Backtracking iterative improvement

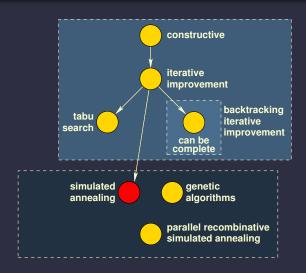
- Allows B steps of backtracking
- Can be incomplete
- Complete if B = the problem decision depth
- Allows use of problem-speficic heuristics for ordering
- Incomplete if B < decision depth</li>
- More on this later

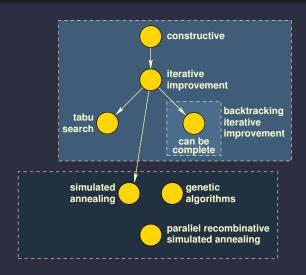


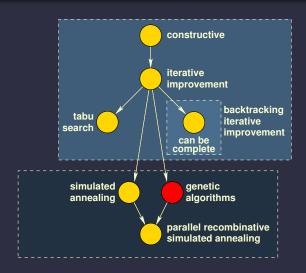


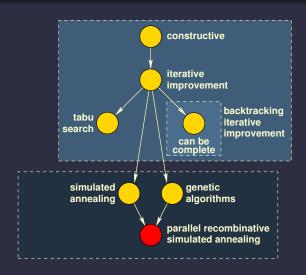












# Constructive algorithms

- Build solution piece by piece
- Once complete solution is generated, don't change
- Typically fast
- Easy to use problem-specific information
- Easy to implement
- Prone to becomming trapped in poor search space







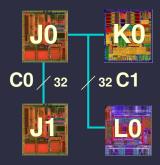


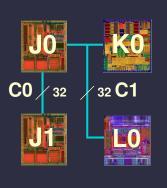


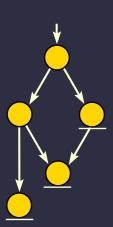


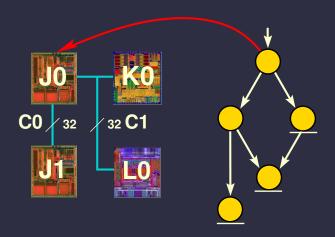


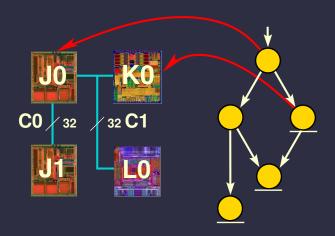


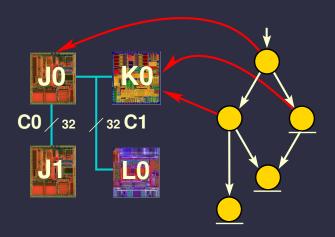


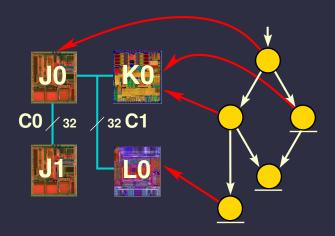


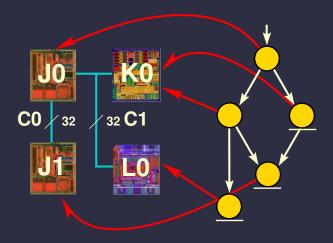








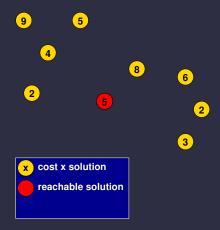


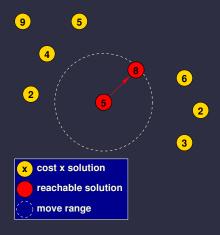


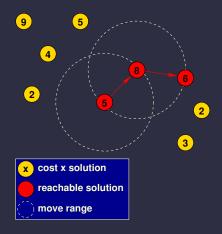
#### Iterative improvement

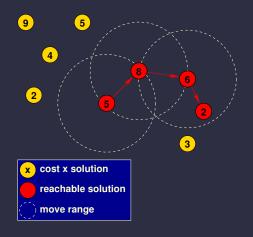
- Starts with complete but poor solution
  - therefore contains constructive algorithm
  - superset of constructive
- Makes changes to solution to improve it
- Typically fast
- Easy to use problem-specific information
- Easy to implement
- Prone to becomming trapped in local minima

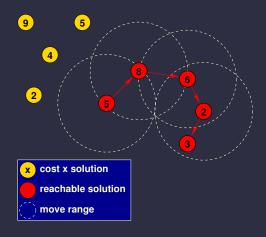
#### Local minima move size

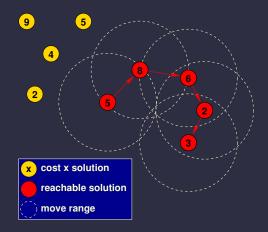


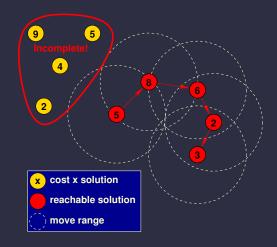


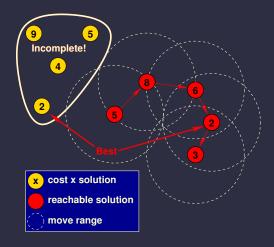












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#### Local minima

- Even if all solutions reachable, may not get best solution
- Depends on move selection













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#### Local mimina

- Being trapped in local minima is a big problem
- Numerous probabilistic optimization techniques designed
  - avoid local minima
  - find global minima
  - do so efficiently

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# Backtracking iterative improvement

- Backtracking iterative improvement is complete if
  - all solutions are reachable
  - the backtracking depth ≥ search depth
  - ...however, this can be slow
- Even if incomplete, backtracking can improve quality
- Can trade optimization time for solution quality
- Greedy iterative improvement if backtracking depth is zero







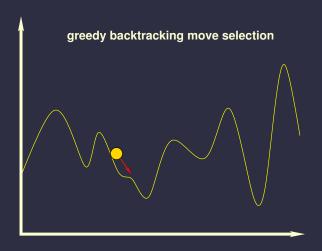
















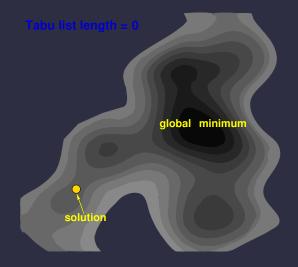


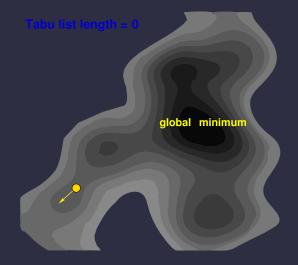


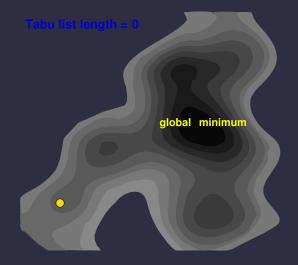
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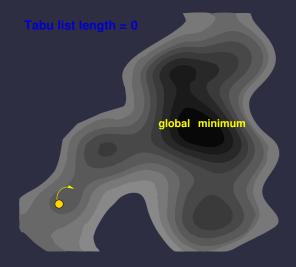
#### Tabu search

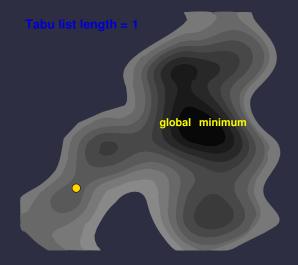
- Similar to interative improvement
- Iterative improvement can cycle
  - chooses largest cost decrease move...
  - ... then chooses smallest cost increase move
- Tabu search has a tabu list
  - solutions to avoid
  - solution characteristics to avoid
- Prevents iterative cycles

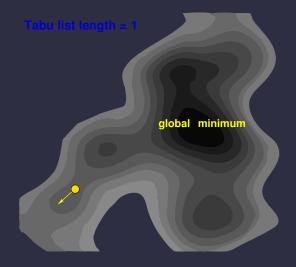


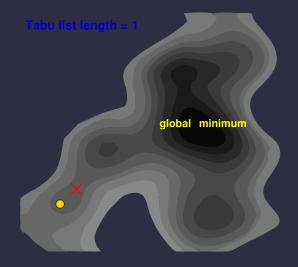


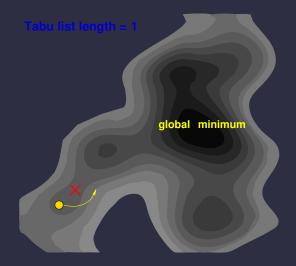


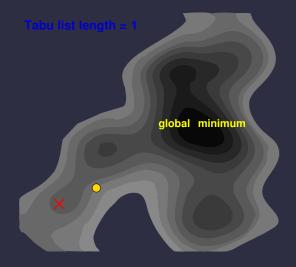


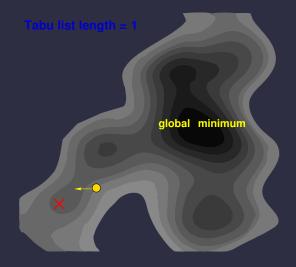


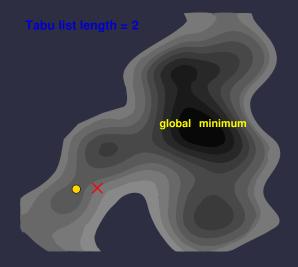


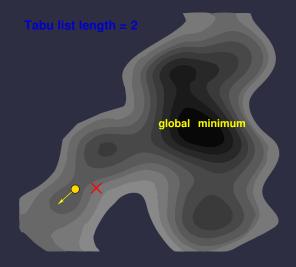


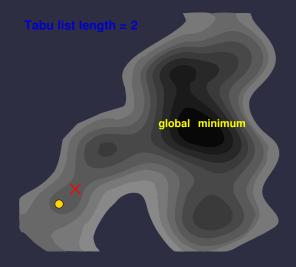


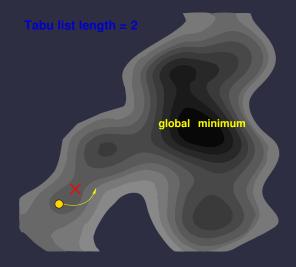


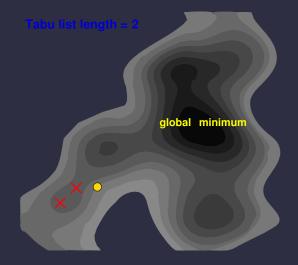


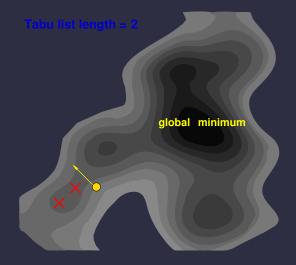


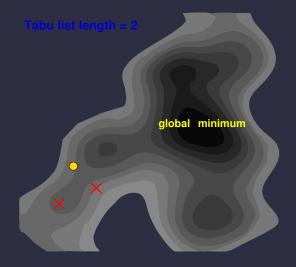


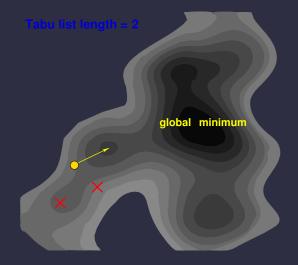


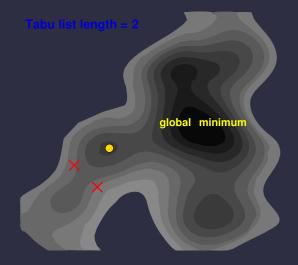


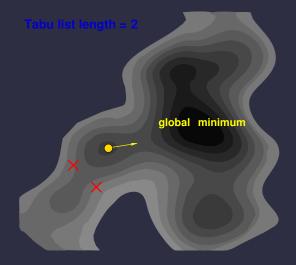


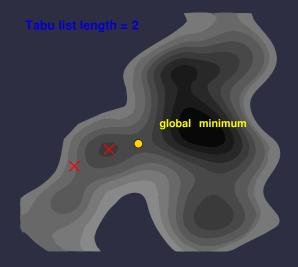


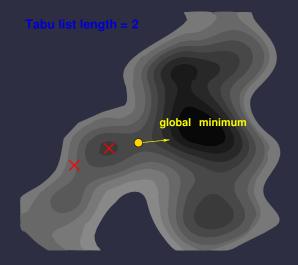


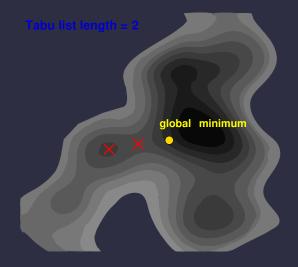


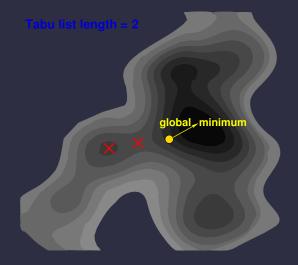


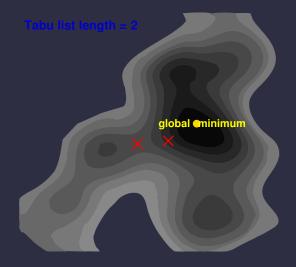


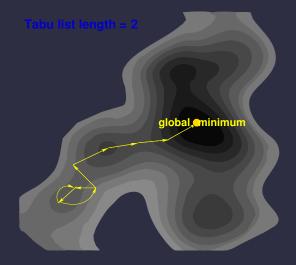












# Simulated annealing

- Inspired by annealing of metals
- Start from high temperature and gradually lower
- Avoids local minima traps
- Generate trial solutions
- Conduct Boltzmann trials between old and new solution

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# Simulated annealing

- Easy to implement
- Can trade optimization time for solutions quality
- Greedy iterative improvement if temperature is zero
- Famous for solving difficult physical problems, e.g., placement

#### Boltzmann trials

Solution are selected for survival by conducting Boltzmann trials between parents and children.

Given a global temperature T, a solution with cost K beats a solution with cost J with probability:

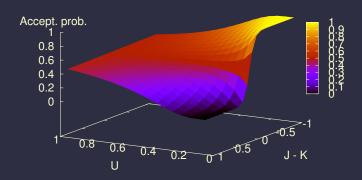
$$\frac{1}{1+e^{(J\text{-}K)/T}}$$

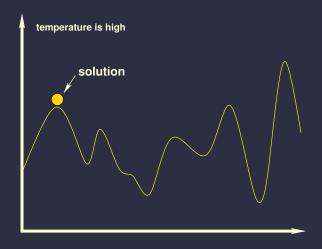
#### Boltzmann trials

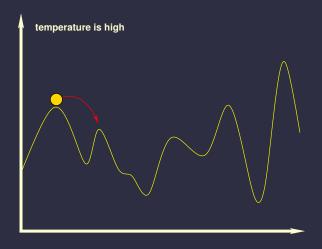
Introduce convenience variable *U* 

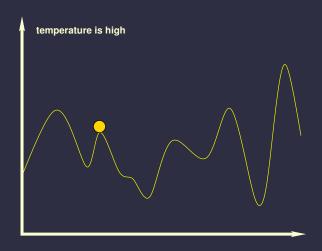
$$egin{aligned} U(T) &= 1 - rac{1}{T+1} \ U(0) &= 0 \ T 
ightarrow 1 \Rightarrow U(T) 
ightarrow \infty \end{aligned}$$

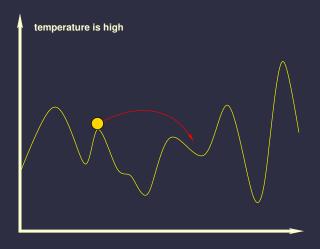
#### Boltzmann trials

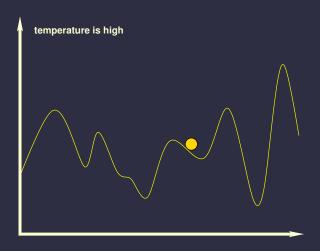


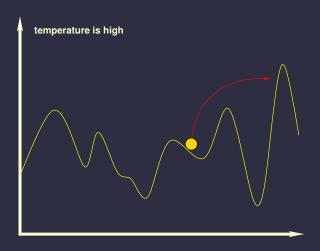


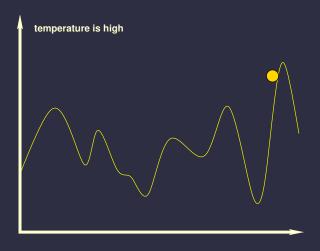


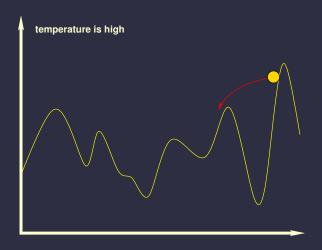


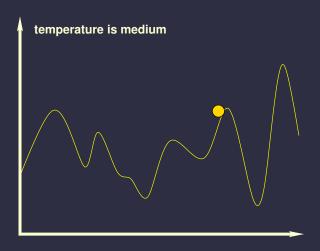


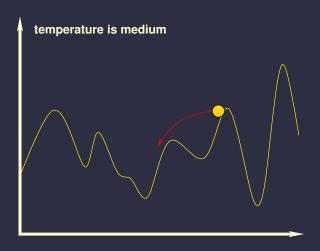


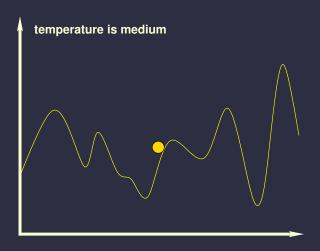


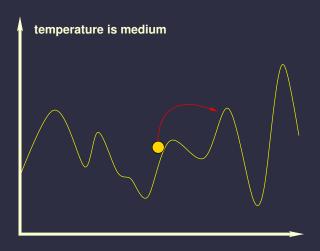


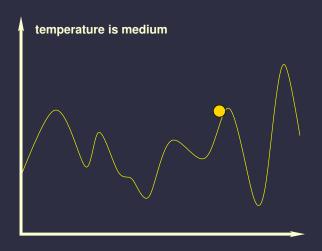


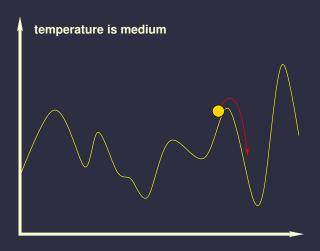


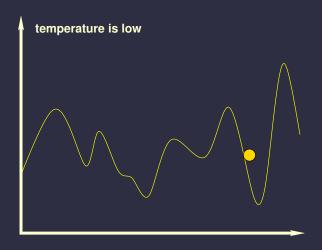


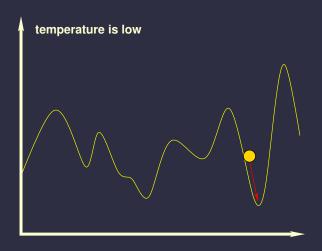


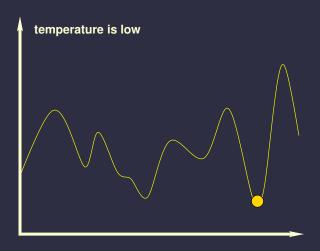


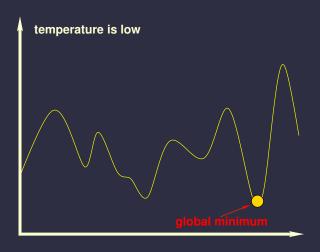


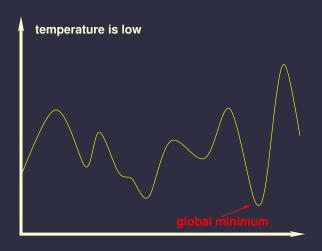




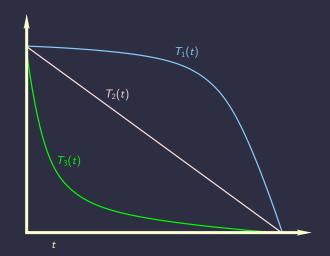








# Cooling schedule often not important

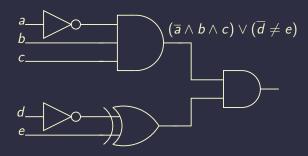


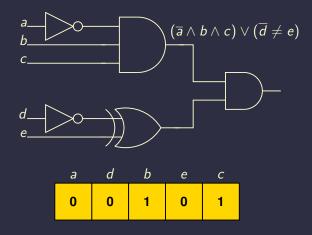
# Simulated annealing notes

- Time complexity extremely difficult to analyze
- Given a slow enough cooling schedule, will get optimum
  - This schedule sometimes makes simulated anealing slower than exhaustive search
  - Determining optimal schedule requires detailed knowledge of problem's Markov chains

# Genetic algorithms

- Multiple solutions
- Local randomized changes to solutions
- Solutions share information with each other
- Can trade optimization time for solution quality
- Good at escaping sub-optimal local minima
- Greedy iterative improvement if no information sharing
- Difficult to implement and analyze
- Researchers have applied in testing, system synthesis

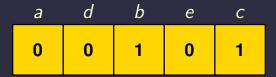




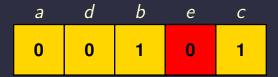
#### Mutation

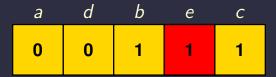
- Choose an element of the solution
- Change it to another value
- Local modification, similar to that in iterative improvement

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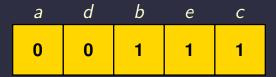


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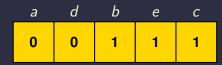




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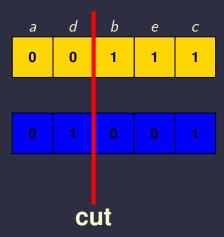


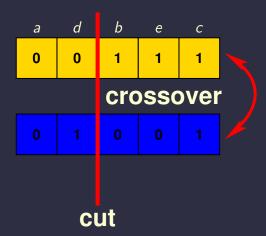
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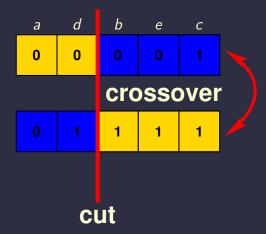


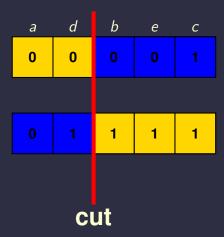
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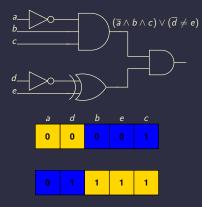


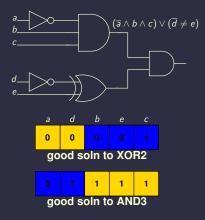


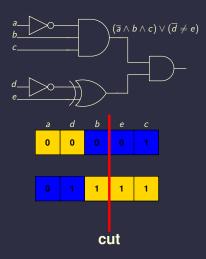


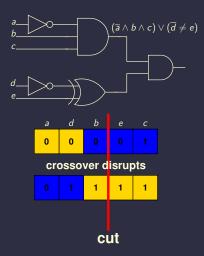
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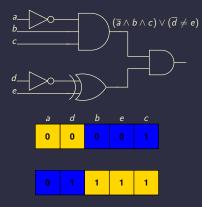


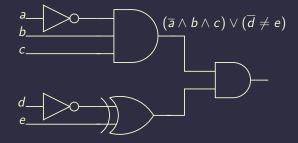


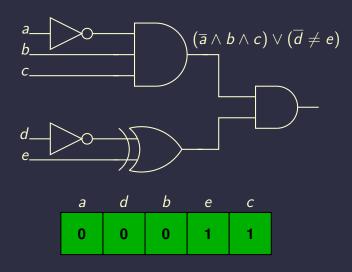


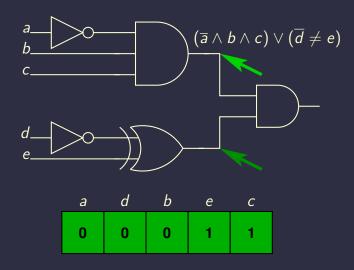


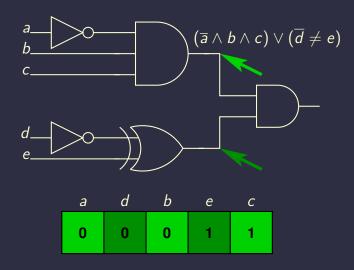




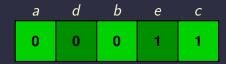


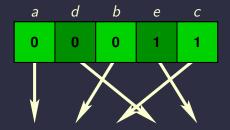


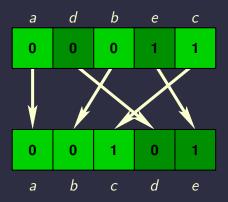


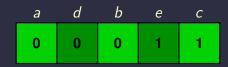


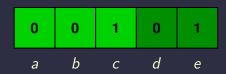
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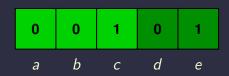


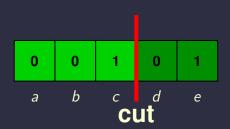


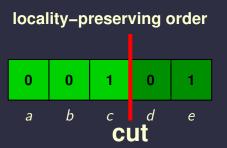




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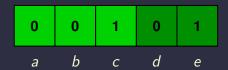




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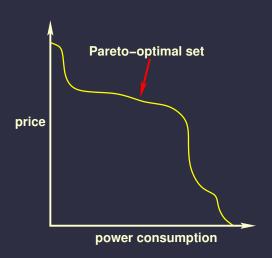
# Locality preserved

# locality-preserving order crossover doesn't disrupt sub-solutions

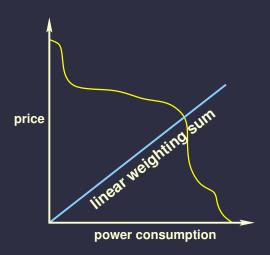


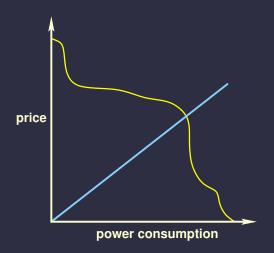
# Multidimensional optimization

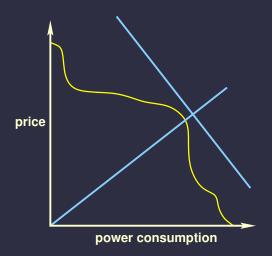
- Real-world problems often have multiple costs
  - Price
  - Power consumption
  - Speed
  - Temperature
  - Reliability
  - etc.
- Necessary to simultaneously minimize all costs



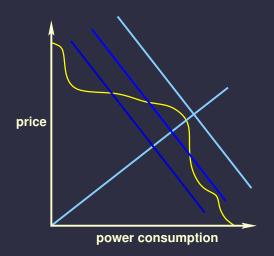




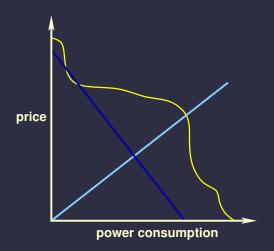


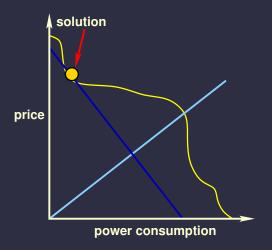










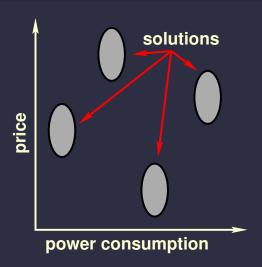


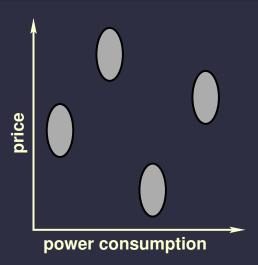
A solution dominates another if all its costs are lower, i.e.,

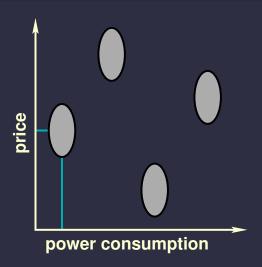
$$\mathsf{dom}_{a,b} = \forall_{i=1}^n cost_{a,i} < cost_{b,i} \land a \neq b$$

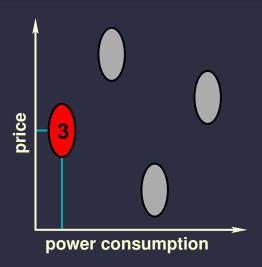
A solution's rank is the number of other solutions which do not dominate it, i.e.,

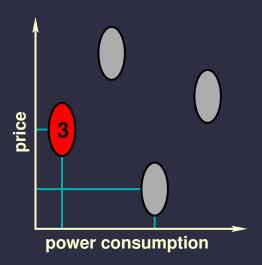
$$\operatorname{rank}_{s'} = \sum_{i=1}^n \operatorname{not} \operatorname{dom}_{s_i,s'}$$

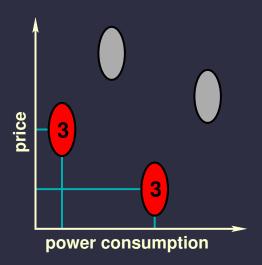


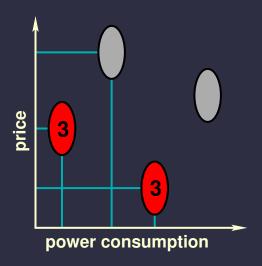


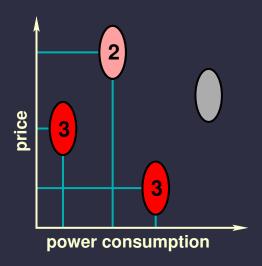


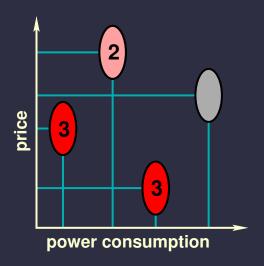


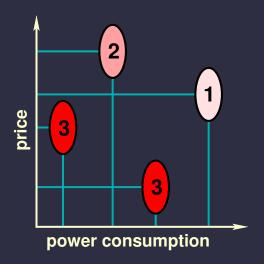






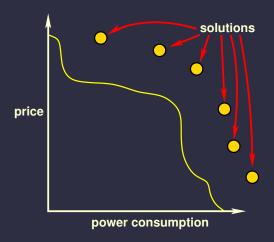




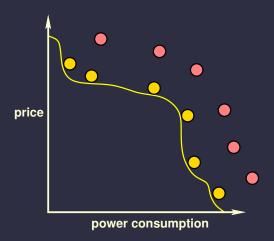












# Genetic algorithm selection

- Solutions are selected for survival by cost or rank
- Resistant to becoming trapped in local minima
  - mutation
  - crossover
- Possible to do better?

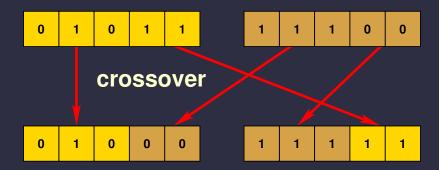
#### PRSA

- Genetic algorithm where Boltzmann trials are used for solution selection
- Genetic algorithm if temperature is set to zero
- Simulated annealing if only one solution
- Easily parallizable
- Has strengths of genetic algorithms and simulated annealing
- Difficult to implement but not more difficult than genetic algorithms

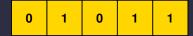
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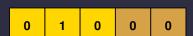
# PRSA example

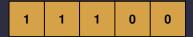


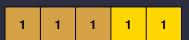


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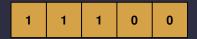
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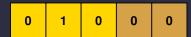


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# Multiobjective GAs

Carlos M. Fonseca and Peter J. Fleming. Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization.

In Proc. Int. Conf. Genetic Algorithms, pages 416-423, July 1993

- Explains importance of multiobjective optimization
- Shows simple way to use Pareto-rank in parallel optimization meta-heuristics

## Very high-level optimization reference

Robert P. Dick. *Multiobjective synthesis of low-power real-time distributed embedded systems.* 

PhD thesis, Dept. of Electrical Engineering, Princeton University, July 2002

- Chapter 4 contains an overview of some of the popular probabilistic optimization techniques used in CAD
- Chapters 5 and 6 describe a PRSA for system synthesis.

## **Evolutionary algorithms**

D. Graham-Rowe. Radio emerges from the electronic soup. *New Scientist*, August 2002

- Interesting short article on a phyical application on evolutionary algorithms
- Similar results for FPGA-based filter

## Genetic algorithms reference

David E. Goldberg. *Genetic Algorithms in Search, Optimization, and Machine Learning*.

Addison-Wesley, MA, 1989

- The most basic and complete book on genetic algorithms
- Weak on multiobjective potential this meta-heuristic

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#### PRSA reference

Samir W. Mahfoud and David E. Goldberg. Parallel recombinative simulated annealing: A genetic algorithm. *Parallel Computing*, 21:1–28, January 1995

#### Outline

- 1. Optimization for synthesis
- 2. Homework

### What to do by Wednesday evening

#### Send the following things to me by email

- An itemized list of 1-3 value propositions, i.e., the values you think your embedded system or research idea can provide to your customers.
- A text file containing a paragraph-long description of the embedded system you are currently planning to prototype.
- A text file listing the 2-3 most important hypotheses you are attempting to validate or invalidate via interviews.
- A text file (or files) containing notes from all your interviews.
   You should have around 10 by Wednesday. Each should contain the following.
  - Date and time.
  - Name of interviewee.
  - Why the interviewee is a potential customer.
  - A chronologically organized series of questions and answers.
     These can be terse