Embedded Systems: An Application-Centered Approach

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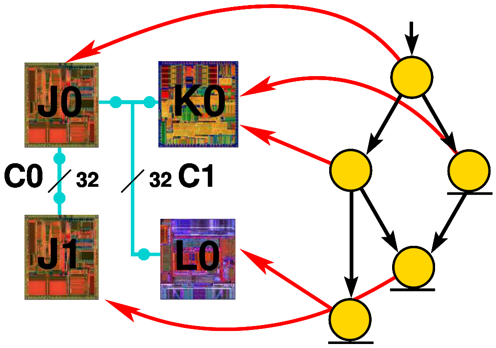
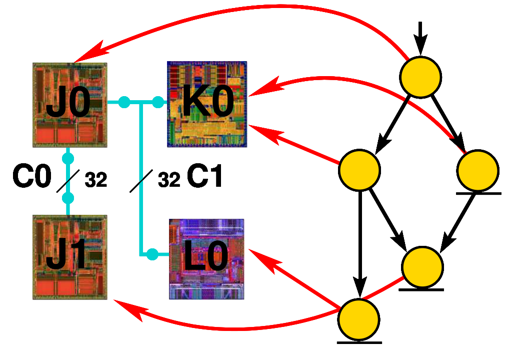
Synthesis motivation

- Embedded systems are found everywhere: cars, houses, games, phones, hospitals, etc.
- Designers need tools to deal with increasing complexity, increase product quality, and guarantee correct operation.
- Software or hardware errors are not acceptable. Anti-lock brake systems aren't allowed to crash.
- Embedded systems should not require bug fixes or upgrades.
- Price competition can be intense.
- Power consumption should be low.

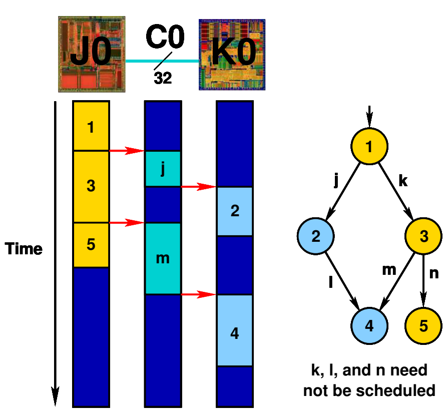
Allocation



Assignment

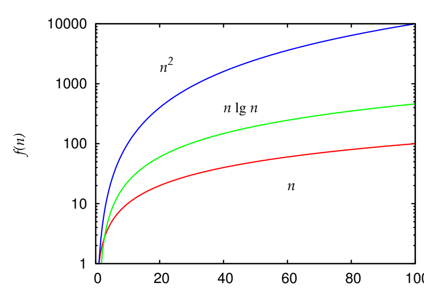


Scheduling



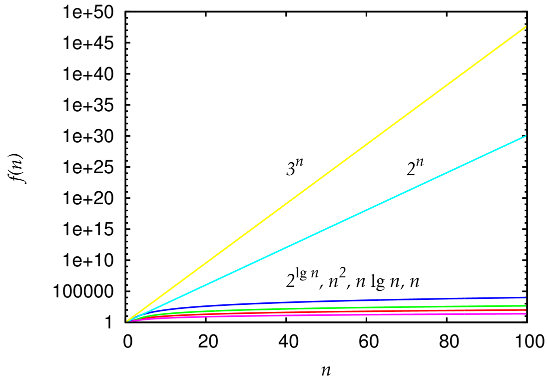
Polynomial time complexities

- Recall that sorting may be done in $\mathcal{O}(n \lg n)$ time
- DFS $\in \mathcal{O}(|V| + |E|)$, BFS $\in \mathcal{O}(|V|)$
- Topological sort $\in \mathcal{O}(|V| + |E|)$



Exponential time complexities

There also exist exponential-time algorithms: $\mathcal{O}\left(2^{\lg n}\right)$, $\mathcal{O}\left(2^n\right)$, $\mathcal{O}\left(3^n\right)$



Implications of exponential time complexity

For $t(n) = 2^n$ seconds

$$
\begin{aligned}
t(1) &= 2 \text{ seconds} \\
t(10) &= 17 \text{ minutes} \\
t(20) &= 12 \text{ days} \\
t(50) &= 35,702,052 \text{ years} \\
t(100) &= 40,196,936,841,331,500,000,000 \text{ years}
\end{aligned}
$$

$\mathcal{NP}$-complete problems

- Digital design and synthesis is full of NP-complete problems
- Graph coloring
- Allocation/assignment
- Scheduling
- Graph partitioning
- Satisfiability (and 3SAT)
- *Covering*
- . . . and many more

Conjecture on hardness of problems

- There is a class of problems, $\mathcal{NP}$-complete, for which nobody has found polynomial time solutions
- It is possible to convert between these problems in polynomial time
- Thus, if it is possible to solve any problem in $\mathcal{NP}$-complete in polynomial time, all can be solved in polynomial time
- $\mathcal{P} \subseteq \mathcal{NP}$
- Unproven conjecture: $\mathcal{P} \neq \mathcal{NP}$

$\mathcal{NP}$

- What is $\mathcal{NP}$? Nondeterministic polynomial time.
- A computer that can simultaneously follow multiple paths in a solution space exploration tree is nondeterministic. Such a computer can solve $\mathcal{NP}$ problems in polynomial time.
- Nobody has been able to prove either

  $$\mathcal{P} \neq \mathcal{NP}$$

  or

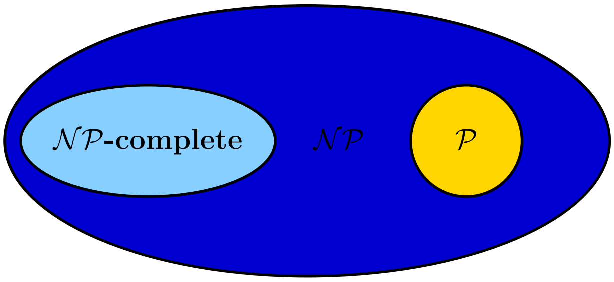
  $$\mathcal{P} = \mathcal{NP}$$

$\mathcal{NP}$-completeness

If we define $\mathcal{NP}$-complete to be a set of problems in $\mathcal{NP}$ for which any problem's instance may be converted to an instance of another problem in $\mathcal{NP}$-complete in polynomial time, then

$$\mathcal{P} \subsetneq \mathcal{NP} \Rightarrow \mathcal{NP}\text{-complete} \cap \mathcal{P} = \varnothing$$

Basic complexity classes



- $\mathcal{P}$ solvable in polynomial time by a computer (Turing Machine).
- $\mathcal{NP}$ solvable in polynomial time by a nondeterministic computer.
- $\mathcal{NP}$-complete converted to other $\mathcal{NP}$-complete problems in polynomial time.

How to deal with hard problems

- What should you do when you encounter an apparently hard problem?
- Is it in $\mathcal{NP}$-complete?
- If not, solve it
- If so, then what?

Despair. Solve it! Resort to a suboptimal heuristic.
Bad, but sometimes the only choice. Develop an approximation algorithm.
Better. Determine whether all encountered problem instances are constrained.
Wonderful when it works.

One example

O. Coudert. Exact coloring of real-life graphs is easy. *Design Automation*, pages 121–126, June 1997.

Properties of complete optimization techniques

- If a solution exists, will be found
- Very slow for some problems
- Good formal understanding of complexity

Example complete algorithms

- Enumeration
- Branch and bound
- Dynamic programming
- Integer-linear programming
- Backtracking iterative improvement

Enumeration

- Considers all possible solutions
- Extremely slow for large *n*
- Potentially has low constant factor, may be O.K. for small *n*

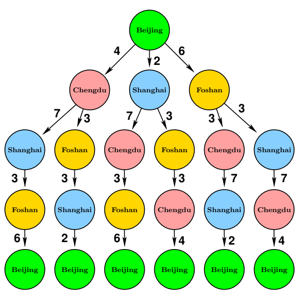
Example problem

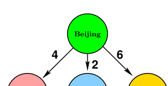
Traveling salesman problem

Find shortest path visiting all cities.

Traveling salesman problem

Enumeration





Branch and bound

- Keep track of minimal encountered cost
- When a path has a higher cost, terminate

Branch and bound

Branch and bound

- Better average-case complexity
- Still worst-case exponential

Linear programming

- In $\mathcal{P}$– Ellipsoid Algorithm / internal point methods
- However, in practice WC exponential Simplex Algorithm better
- Goal: Maximize a linear weighted sum under constraints on variables

Linear programming

Maximize

$$c_1 \cdot x_1 + c_2 \cdot x_2 + \cdots + c_n \cdot x_n$$

where

$$\forall c_i \in c, c_i \in R$$

subject to the following constraints:

$$\begin{array}{ll} a_{11} \cdot x_1 + a_{12} \cdot x_2 + \cdots + a_{1n} \cdot x_n \leq,=,\geq & b_1 \\ a_{21} \cdot x_1 + a_{22} \cdot x_2 + \cdots + a_{2n} \cdot x_n \leq,=,\geq & b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \cdots \\ a_{n1} \cdot x_1 + a_{1n} \cdot x_2 + \cdots + a_{nn} \cdot x_n \leq,=,\geq & b_n \\ \forall x_i \in x, x_i \geq 0 \qquad\qquad \forall a_{jk} \in A, a_{jk} \in R & \end{array}$$

Linear programming

- Can be formulated as a linear algebra problem
  - Vector x of variables
  - Vector c of cost
  - Matrix A of constraints
  - Vector b of constraints
- Maximize or minimize $c^T x$
- Satisfy $Ax \leq b$
- Satisfy $x \geq 0$

Integer-linear programming (ILP)

- ILP is $\mathcal{NP}$-complete
- LP with some variables restricted to integer values
- Formulate problem as ILP problem
  - Excellent understanding of problem
  - Good solvers exist
- Variants – both NP-complete
  - Mixed ILP has some continuous variables
  - Zero-one ILP

Example – ILP formulation for the travaling salesman problem

Let T be a tentative solution, or tour
$\forall e \in E$ let there be a variable

$$t_e = \begin{cases} 1 & \text{if } e \in T \\ 0 & \text{if } e \notin T \end{cases}$$

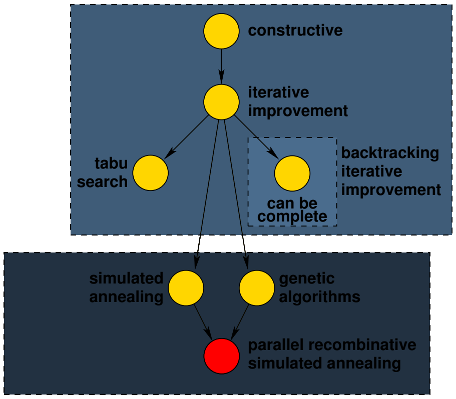
Constraint: Given that S is a set of vertices, $\mathbf{con}(S)$ is the set of edges connecting $v \in S$ to $v \notin S$, and $\{v_i\}$ is the vertex set containing only v_i, every vertex, v_i must be connected to two edges of the tour

$$\forall v_i \in V, \sum_{e \in \mathbf{con}(\{v_i\})} = 2$$

Backtracking iterative improvement

- Allows B steps of backtracking
- Can be incomplete
- Complete if $B =$ the problem decision depth
- Allows use of problem-speficic heuristics for ordering
- Incomplete if $B <$ decision depth
- More on this later

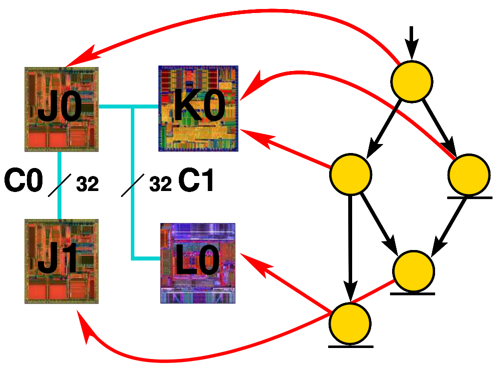
Optimization techniques



Constructive algorithms

- Build solution piece by piece
- Once complete solution is generated, don't change
- Typically fast
- Easy to use problem-specific information
- Easy to implement
- Prone to becomming trapped in poor search space

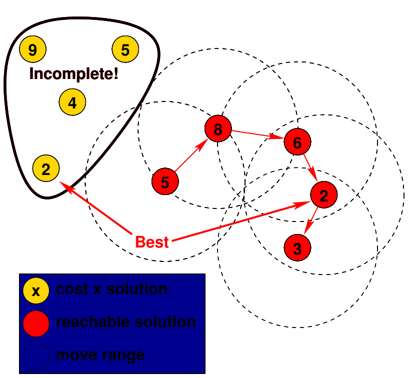
Constructive algorithms example



Iterative improvement

- Starts with complete but poor solution
  - therefore contains constructive algorithm
  - superset of constructive
- Makes changes to solution to improve it
- Typically fast
- Easy to use problem-specific information
- Easy to implement
- Prone to becomming trapped in local minima

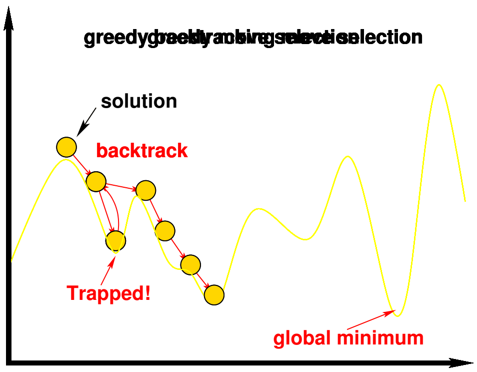
Local minima move size



Local minima

- Even if all solutions reachable, may not get best solution
- Depends on move selection

Trapped in local minima



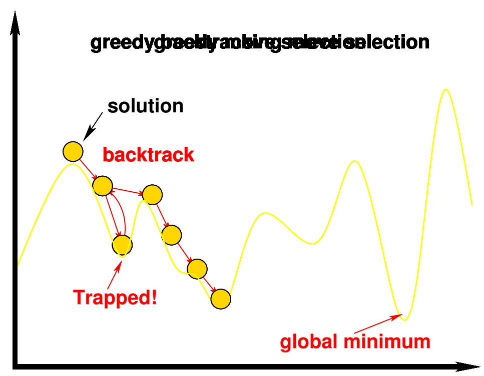
Local mimina

- Being trapped in local minima is a big problem
- Numerous probabilistic optimization techniques designed
  - avoid local minima
  - find global minima
  - do so efficiently

Backtracking iterative improvement

- Backtracking iterative improvement is complete if
  - all solutions are reachable
  - the backtracking depth $\geq$ search depth
  - . . . however, this can be slow
- Even if incomplete, backtracking can improve quality
- Can trade optimization time for solution quality
- Greedy iterative improvement if backtracking depth is zero

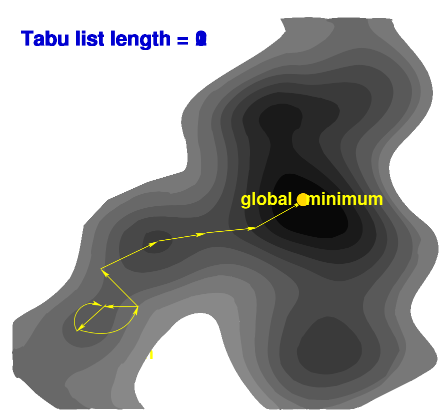
Backtracking



Tabu search

- Similar to interative improvement
- Iterative improvement can cycle
  - chooses largest cost decrease move. . .
  - . . . then chooses smallest cost increase move
- Tabu search has a *tabu list*
  - solutions to avoid
  - solution characteristics to avoid
- Prevents iterative cycles

Tabu search example



Simulated annealing

- Inspired by annealing of metals
- Start from high temperature and gradually lower
- Avoids local minima traps
- Generate trial solutions
- Conduct Boltzmann trials between old and new solution

Simulated annealing

- Easy to implement
- Can trade optimization time for solutions quality
- Greedy iterative improvement if temperature is zero
- Famous for solving difficult physical problems, e.g., placement

Boltzmann trials

Solution are selected for survival by conducting Boltzmann trials between parents and children.
Given a global temperature T, a solution with cost K beats a solution with cost J with probability:

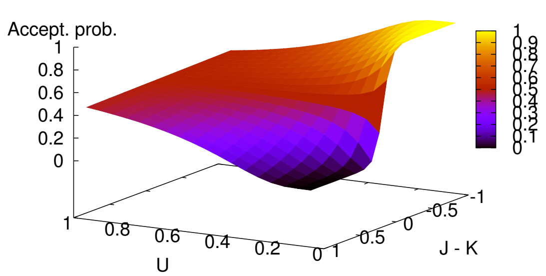
$$\frac{1}{1 + e^{(J\text{-}K)/T}}$$

Boltzmann trials

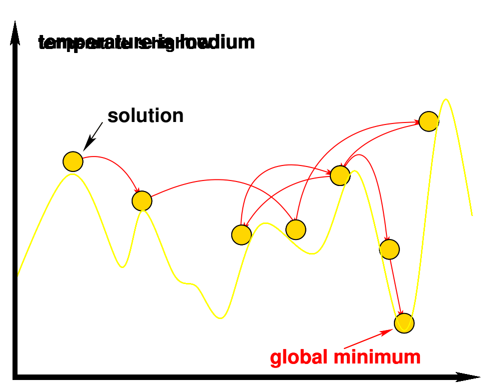
Introduce convenience variable U

$$U(T) = 1 - \frac{1}{T+1}$$
$$U(0) = 0$$
$$T \to 1 \Rightarrow U(T) \to \infty$$

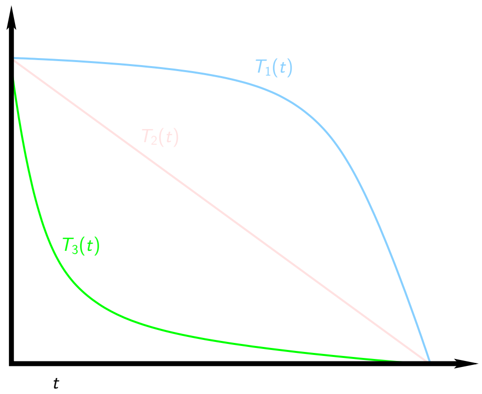
Boltzmann trials



Simulated annealing example



Cooling schedule often not important



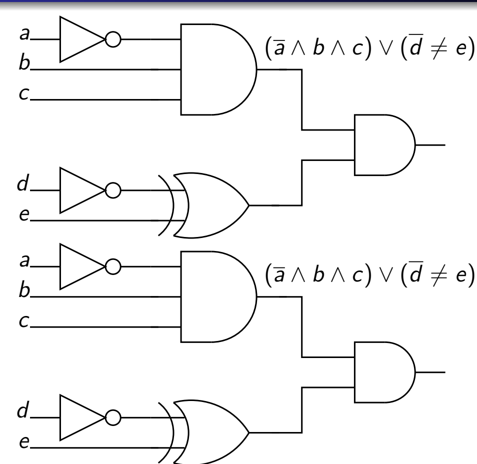
Simulated annealing notes

- Time complexity extremely difficult to analyze
- Given a slow enough cooling schedule, will get optimum
  - This schedule sometimes makes simulated anealing slower than exhaustive search
  - Determining optimal schedule requires detailed knowledge of problem's Markov chains

Genetic algorithms

- Multiple solutions
- Local randomized changes to solutions
- Solutions share information with each other
- Can trade optimization time for solution quality
- Good at escaping sub-optimal local minima
- Greedy iterative improvement if no information sharing
- Difficult to implement and analyze
- Researchers have applied in testing, system synthesis

Solution representation



Mutation

- Choose an element of the solution
- Change it to another value
- Local modification, similar to that in iterative improvement

Solution representation

a	d	b	e	c
0	**0**	**1**	**0**	**1**

a	d	b	e	c
0	**0**	**1**	**0**	**1**

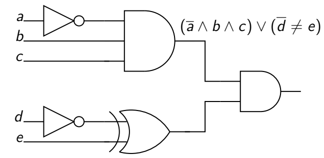
a	d	b	e	c
0	**0**	**1**	**1**	**1**

a	d	b	e	c
0	**0**	**1**	**1**	**1**

Crossover

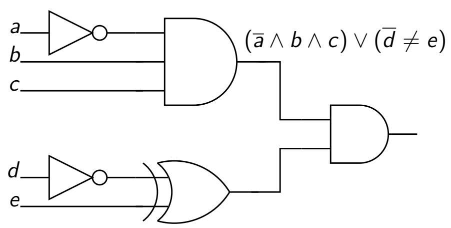
a	d	b	e	c
0	0	1	1	1

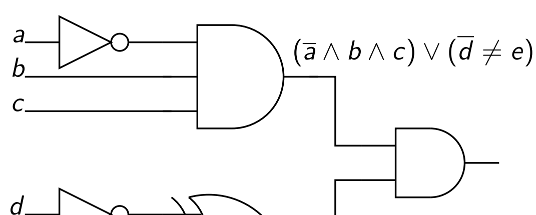
Locality not preserved



a	d	b	e	c
0	0	0	0	1
0	1	1	1	1

Locality preserved

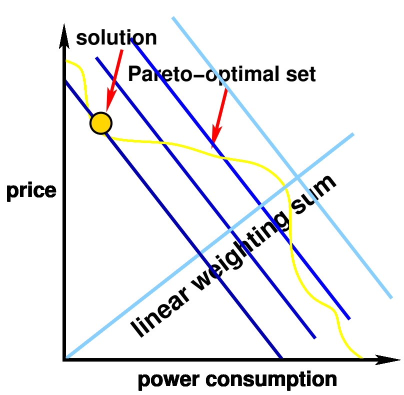




Multidimensional optimization

- Real-world problems often have multiple costs
  - Price
  - Power consumption
  - Speed
  - Temperature
  - Reliability
  - etc.
- Necessary to simultaneously minimize all costs

Linear weighting sum



Pareto-ranking

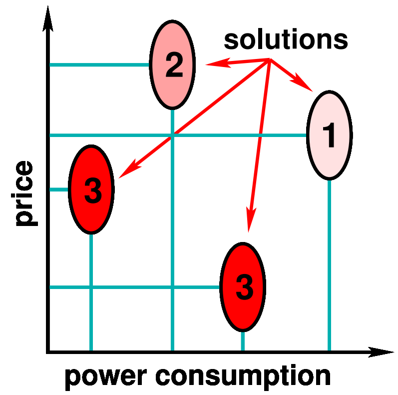
A solution dominates another if all its costs are lower, i.e.,

$$\mathbf{dom}_{a,b} = \forall_{i=1}^{n} cost_{a,i} < cost_{b,i} \wedge a \neq b$$

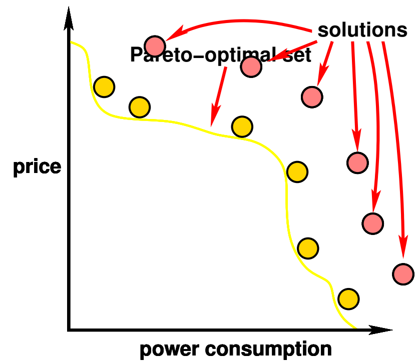
A solution's rank is the number of other solutions which do not dominate it, i.e.,

$$\mathbf{rank}_{s'} = \sum_{i=1}^{n} \mathbf{not\ dom}_{s_i,s'}$$

Pareto-ranking



Pareto-rank based multiobjective optimization



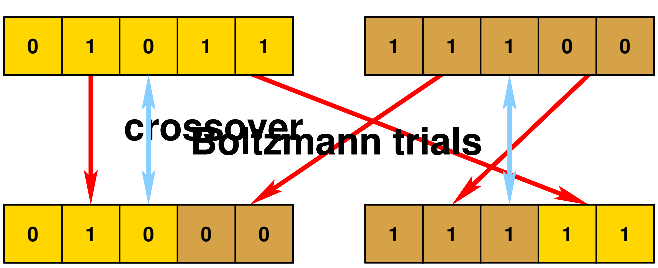
Genetic algorithm selection

- Solutions are selected for survival by cost or rank
- Resistant to becoming trapped in local minima
  - mutation
  - crossover
- Possible to do better?

PRSA

- Genetic algorithm where Boltzmann trials are used for solution selection
- Genetic algorithm if temperature is set to zero
- Simulated annealing if only one solution
- Easily parallelizable
- Has strengths of genetic algorithms and simulated annealing
- Difficult to implement but not more difficult than genetic algorithms

PRSA example



Multiobjective GAs

Carlos M. Fonseca and Peter J. Fleming. Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization.
In *Proc. Int. Conf. Genetic Algorithms*, pages 416–423, July 1993

- Explains importance of multiobjective optimization
- Shows simple way to use Pareto-rank in parallel optimization meta-heuristics

Very high-level optimization reference

Robert P. Dick. *Multiobjective synthesis of low-power real-time distributed embedded systems.*
PhD thesis, Dept. of Electrical Engineering, Princeton University, July 2002

- Chapter 4 contains an overview of some of the popular probabilistic optimization techniques used in CAD
- Chapters 5 and 6 describe a PRSA for system synthesis.

Evolutionary algorithms

D. Graham-Rowe. Radio emerges from the electronic soup.
New Scientist, August 2002

- Interesting short article on a phyical application on evolutionary algorithms
- Similar results for FPGA-based filter

Genetic algorithms reference

David E. Goldberg. *Genetic Algorithms in Search, Optimization, and Machine Learning.*
Addison-Wesley, MA, 1989

- The most basic and complete book on genetic algorithms
- Weak on multiobjective potential this meta-heuristic

PRSA reference

Samir W. Mahfoud and David E. Goldberg. Parallel recombinative simulated annealing: A genetic algorithm.
Parallel Computing, 21:1–28, January 1995

What to do by Wednesday evening

Send the following things to me by email

- An itemized list of 1-3 value propositions, i.e., the values you think your embedded system or research idea can provide to your customers.
- A text file containing a paragraph-long description of the embedded system you are currently planning to prototype.
- A text file listing the 2-3 most important hypotheses you are attempting to validate or invalidate via interviews.
- A text file (or files) containing notes from all your interviews. You should have around 10 by Wednesday. Each should contain the following.
  - Date and time.
  - Name of interviewee.
  - Why the interviewee is a potential customer.
  - A chronologically organized series of questions and answers. These can be terse.